

Closed Form Solution for QoS–Constrained Information–Theoretic Sum Capacity of Reverse Link CDMA Systems

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ABSTRACT

The information-theoretic maximum system capacity in a CDMA network, which guarantees fairness to mobile stations, can be achieved by optimally allocating transmit powers to the mobile stations while imposing individual QoS constraints. Even though this optimization is carried out in a multi-dimensional space we show that the optimal solution can be reduced to a search in a one-dimensional space. In this paper we first show that the previous results can be obtained by a simpler and more intuitive approach. Also, we show that the optimal solution is actually bang–bang. In addition, we show that the candidate solutions can be determined in a closed form. This considerably reduces the computational effort. Because rather than carrying out numerical search on a set of intervals, only a few explicitly determined points should now be examined.

Categories and Subject Descriptors

H.1 [Models and Principles]: General

General Terms

Management

Keywords

Quality of Service, Reverse Link CDMA, Optimization

1. INTRODUCTION

In a recent paper *Oh* and *Soong* [6, 7] developed a method for obtaining reverse link capacity of a CDMA system under some imposed quality of service constraints. In the current paper we improve the results by significantly reducing the computational effort required.

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Capacity of a single point–to–point communication link is well approximated by the well–known Shannon theorem as $C = B \log_2(1 + S/N)$ [3] (Also, see [5]). Here, B is the bandwidth and S/N is the signal to noise ratio of the communication link. Assume that there are M mobile stations whose reverse link gains are g_1, \dots, g_M , satisfying $g_1 > \dots > g_M$. Define the i -th mobile station's transmit power as p_i , for which we have $0 \leq p_i \leq p_{max}$. With a background noise of I , the signal to noise ratio for the i -th station becomes,

$$\left(\frac{S}{N}\right)_i = \frac{p_i g_i}{I + \sum_{j=1, j \neq i}^M p_j g_j}. \quad (1)$$

Substituting (1) into the Shannon formula we have,

$$C_i = \log_2 \frac{I + \sum_{j=1}^M p_j g_j}{I + \sum_{j=1, j \neq i}^M p_j g_j}, \quad (2)$$

which results in the aggregate capacity of,

$$C(\vec{p}) = \sum_{i=1}^M C_i = \log_2 \frac{\left(I + \sum_{j=1}^M p_j g_j\right)^M}{\prod_{i=1}^M \left(I + \sum_{j=1, j \neq i}^M p_j g_j\right)}, \quad (3)$$

where, $\vec{p} = (p_1, \dots, p_M)$.

There are some conditions which should be added to the problem. First, the total power reaching the base station should be smaller than a fixed margin ($\sum_{i=1}^M p_i g_i \leq P_{max}$). Also, each station should have a minimum threshold quality of service ($\forall i, (S/N)_i \geq \gamma$). The above formulations lead to the optimization problem stated as finding $\vec{p} = (p_1, \dots, p_M)$, which,

$$\max_{\vec{p}} \frac{\left(I + \sum_{j=1}^M p_j g_j\right)^M}{\prod_{i=1}^M \left(I + \sum_{j=1, j \neq i}^M p_j g_j\right)}, \quad (4)$$

$$\forall i, 0 \leq p_i \leq p_{max}, \quad (5)$$

$$\forall i, \frac{p_i g_i}{I + \sum_{j=1, j \neq i}^M p_j g_j} \geq \gamma, \quad (6)$$

$$\sum_{i=1}^M p_i g_i \leq P_{max}. \quad (7)$$

Analysis of this problem is given in [6, 7]. There, the authors derive useful results and limit the search–space from R^M to

M one-dimensional intervals in R . The algorithm given in [6, 7] uses numerical methods at its core. In this way, a number of intervals, of order M , should be numerically searched for the best solution. In this paper, we enhance the results by limiting the search to order of M points in R^M which are given in closed form.

2. PROPOSED METHOD

In this section, the classical problem as introduced in [6, 7] is analyzed. First we carry out a few linear transformations to simplify problem formulation in Section 2.1. Then, we investigate the behavior of the problem in specific hyperplanes and give an important theorem in Section 2.2. In Section 2.3, we localize the solution to a smaller search space than what was developed before and this gives practical inequalities. Then, in Section 2.4 we rigorously analyze the problem and prove another important theorem which is used in Section 2.5 to propose a new algorithm for accurately finding the solution to the problem.

2.1 Linear Transformation of the Search-Space

The first contribution of this paper is to use few transformations to give a simpler representation of the problem. This will also lead to a simpler solution. First, we rewrite (6) as,

$$\forall i, \frac{p_i g_i}{I + \sum_{j=1}^M p_j g_j} \geq \varphi. \quad (8)$$

Here, φ is a fixed parameter defined as $\varphi = \gamma/(\gamma + 1)$. Clearly, $0 < \varphi < 1$. Now, substituting,

$$x_i = \frac{p_i g_i}{I}, \quad (9)$$

into (4) and finding the reciprocal we have to minimize,

$$\Phi(\vec{x}) = \frac{\prod_{i=1}^M (1 + \sum_{j=1}^M x_j - x_i)}{(1 + \sum_{j=1}^M x_j)^M}, \quad (10)$$

where $\vec{x} = (x_1, \dots, x_M)$. Now, we find the new representations of the constraints using the transformation shown in (9). Substituting (9) into (5) gives,

$$\forall i, 0 \leq x_i \leq l_i, l_i = \frac{p_{max} g_i}{I}. \quad (11)$$

Note that as g_i s are sorted in a descending fashion, so are l_i s. Here, to comply with the history of the problem, we have assumed that p_{max} is fixed for all stations. Though, we have never used it, except for proving that l_i s are sorted. Obviously, we may assume that the stations are sorted in the way that $p_{max,i} g_i$ s are ordered in a descending fashion, not g_i s. Hence, the solution given here is more general than the previous discussion given in [6, 7]. Here, $p_{max,i}$ is the maximum transmit power of the i -th station. Substituting (9) into (8) gives,

$$\forall i, \frac{x_i}{1 + \sum_{j=1}^M x_j} \geq \varphi. \quad (12)$$

Finally, (7) becomes,

$$\sum_{i=1}^M x_i \leq X_{max}, X_{max} = \frac{P_{max}}{I}. \quad (13)$$

2.2 Working on a Specific Hyper-Plane

The second contribution of this paper is to investigate the behavior of the problem in specific hyperplanes. This investigation leads to a very important theorem about the problem at the end of this section. Hence, we investigate the problem in the hyperplane,

$$\sum_{i=1}^M x_i = T, \quad (14)$$

for different values of T . In this hyperplane, the problem reduces to finding the minimum of $\Phi(\vec{x}) = (1+T)^{-M} \Phi_T(\vec{x})$, where,

$$\Phi_T(\vec{x}) = \prod_{i=1}^M (1 + T - x_i). \quad (15)$$

While T is fixed, the problem which should be solved is to minimize $\Phi_T(\vec{x})$. On the other hand, the problem can be rephrased as finding T for which minimum value of $\Phi_T(\vec{x})$ is the smallest possible.

In the hyperplane indicated in (14), the constraints have simpler formulations. Inequality (13) simply changes to a limitation for T as,

$$T \leq X_{max}. \quad (16)$$

Also, (12) changes to $\forall i, x_i \geq \varphi(1 + T)$, which, in combination with (11) gives,

$$\forall i, \varphi(1 + T) \leq x_i \leq l_i. \quad (17)$$

Now, the question is to minimize (15) given that \vec{x} lies on the hyperplane defined as (14). The vector \vec{x} should also satisfy (17). Also, T should not become larger than X_{max} . It is clear that if the \vec{x} satisfying (14) is the minimum of (10) then it is the minimum of (15), but not vice versa. Now, we will develop further tools to introduce the essential theorem.

Assume that \vec{x} is the minimum of (15). Also, assume two distinct values of i and j . We will find an important condition about x_i and x_j . Let us rewrite (15) in terms of x_i and x_j . We have, $\Phi_T(\vec{x}) = (1 + T - x_i)(1 + T - x_j) \prod_{k=1, k \neq i, j}^M (1 + T - x_k)$. Here, the last term (which contains $M - 2$ factors) depends neither on x_i nor on x_j . Hence, we have to maximize the product of the first and the second terms. As the summation of these two terms is constant, algebraic operations prove that they should be as much as possible far apart. Using this, we state the essential theorem.

Essential Theorem: In the solution to the problem, there can be no two x_i and x_j s that can increase their distance. We call them an extending joint if they can do so and the situation is called an extension. In other words, the inequalities for x_i s (see (17)) should always stop x_i s from increasing the distance among themselves.

We will use a graphical visualization to explain this rule. Before that, remember a rule of thumb: $\Phi(\vec{x})$ is independent of permutations of x_i s. Also, rather than the bounds for T in (16), the only conditions are those put on x_i s independently (see (17)). Though, in these constraints, the minimum possible value of all x_i s are identical, while maximum possible values are l_i s which are sorted in a descending scheme. Hence, it is reasonable to show the range of x_i as,

$$x_i : [\varphi(1 + T) \dots \dots \dots \leftarrow \times \rightarrow \dots \dots \dots l_i], \quad (18)$$

where, \times shows approximate place of x_i and arrows indicate possible movements. Here, we give names for three situations of x_i . x_i is called to be at the beginning, at the middle, or at the end if $x_i = \varphi(1+T)$, $\varphi(1+T) < x_i < l_i$, or $x_i = l_i$, respectively. Now, to show the power of the essential theorem and the proposed visual notation, we prove the first lemma in [7]. The Lemma says that in the optimum solution there is at most one x_i at the middle. We will tighten this lemma later by showing that actually there is no x_i in the middle, i.e. there is no in-between point. Let us assume otherwise, so that there are x_i and x_j , both at the middle and $i < j$. There are two possibilities, $x_i \geq x_j$ or $x_i < x_j$. The first case leads to,

$$\begin{array}{l} x_i : \left[\varphi(1+T) \cdots \times \rightarrow \cdots l_i \right] \\ x_j : \left[\varphi(1+T) \cdots \leftarrow \times \cdots l_j \right] \end{array},$$

which is clearly an extension. The later case is similar,

$$\begin{array}{l} x_i : \left[\varphi(1+T) \cdots \leftarrow \times \cdots l_i \right] \\ x_j : \left[\varphi(1+T) \cdots \times \rightarrow \cdots l_j \right] \end{array}.$$

We compare this visually intuitive and simple proof with the one given in [6, 7].

The second lemma in [7] states that in the optimum solution, if there is one x_i at the middle, all x_j s for $j > i$ will be at the beginning. Again assume two distinctive x_i and x_j s, where $i < j$ and x_i is at the middle. Now, if x_j is the end, there are two possibilities, namely $x_i \geq l_j$ or $x_i < l_j$. The first one is shown as,

$$\begin{array}{l} x_i : \left[\varphi(1+T) \cdots \times \rightarrow \cdots l_i \right] \\ x_j : \left[\varphi(1+T) \cdots \leftarrow \times l_j \right] \end{array},$$

which is clearly extendable. The second condition is extendable, though a bit harder to realize,

$$\begin{array}{l} x_i : \left[\varphi(1+T) \cdots \times \cdots l_i \right] \\ x_j : \left[\varphi(1+T) \cdots \times l_j \right] \end{array}.$$

Replacing x_i and x_j gives,

$$\begin{array}{l} x_i : \left[\varphi(1+T) \cdots \times \rightarrow \cdots l_i \right] \\ x_j : \left[\varphi(1+T) \cdots \leftarrow \times \cdots l_j \right] \end{array},$$

where the extension is trivial.

Lemma 3 in [7] is talking about x_i s above the one at the middle. The claim is that if x_k is at the middle and for $m < k$, x_m is at the beginning, then for $m < i < k$, x_i can not be at the end. Using the above rules, the result is that x_i should always be at the beginning. The situation which this lemma refers to is,

$$\begin{array}{l} x_m : \left[\varphi(1+T) \times \cdots \cdots l_m \right] \\ x_i : \left[\varphi(1+T) \cdots \cdots \times l_i \right] \\ x_k : \left[\varphi(1+T) \cdots \times \cdots l_k \right], \end{array}$$

which changes to the extendable case of,

$$\begin{array}{l} x_m : \left[\varphi(1+T) \cdots \cdots \times \rightarrow \cdots l_m \right] \\ x_i : \left[\varphi(1+T) \times \cdots \cdots l_i \right] \\ x_k : \left[\varphi(1+T) \cdots \leftarrow \times \cdots l_k \right]. \end{array}$$

Now, like the proposition 1 given in [7] we concludes if there is a solution to the problem then it is as,

$$\vec{x} = \left(l_1, \cdots, l_{k-1}, x_k, \varphi(1+T), \cdots, \varphi(1+T) \right). \quad (26)$$

2.3 Finding Necessary and Sufficient Inequalities for x_k

Equation (26) gives the structure of the optimum solution to the problem. In this section we reduce the size of the search space by finding both necessary and sufficient bounds for both k and x_k . The problem now is to find k and x_k for which the solution given in (26) is optimum. Derivation shows that $T = x_k + L + (M-k)\varphi(1+T)$, where $L = \sum_{i=1}^{k-1} l_i$. Hence,

$$1 + T = \frac{x_k + L + 1}{1 - (M-k)\varphi}. \quad (27)$$

Algebraic operations on the constraints of the problem show that all the conditions are abbreviated in two bounds on x_k ,

$$x_k \leq \min \left\{ l_k, (X_{max} + 1) \left[1 - (M-k)\varphi \right] - L - 1, \right. \quad (28)$$

$$\left. l_M \frac{1 - (M-k)\varphi}{\varphi} - L - 1 \right\},$$

$$x_k \geq \frac{\varphi(L+1)}{1 - (M-k+1)\varphi}, \quad (29)$$

for those values of k satisfying,

$$M - \frac{1}{\varphi} + 1 < k < \frac{1}{\varphi} - \frac{1}{l_M} + 1. \quad (30)$$

2.4 Accurate Spotting of x_k

It was proved in Section 2.2 that the solution to the problem (given in (10)) with the constraints shown in (11), (12), and (13) is like (26). Now, the question is to find appropriate values of k and x_k . Here, we assume that we have selected a value of k and will give practical choices for x_k . In this situation we know from what was described in Section 2.3 that it is both necessary and sufficient for x_k to satisfy (28) and (29). In this section we prove that $\Phi(\vec{x})$ has a very useful behavior in terms of x_k (when other x_i s are selected as in (26)). Actually, we show how to prove that the minimum value of $\Phi(\vec{x})$ happens on a boundary (at a margin). This directly leads to the two choices given in (28) and (29) for x_k . In this way, the problem reduces to checking two values for x_k for any value of k (also limited by (30)). Hence, the solution is actually a bang-bang one, compared to the semi-bang-bang solution developed in [6, 7]. The importance of this non-iterative method over the numerical one used in previous works is clear. Here we have the exact solution to the problem in the proposed method while the available approach is only capable of giving the solution up to some precision. And also, the proposed method is essentially faster.

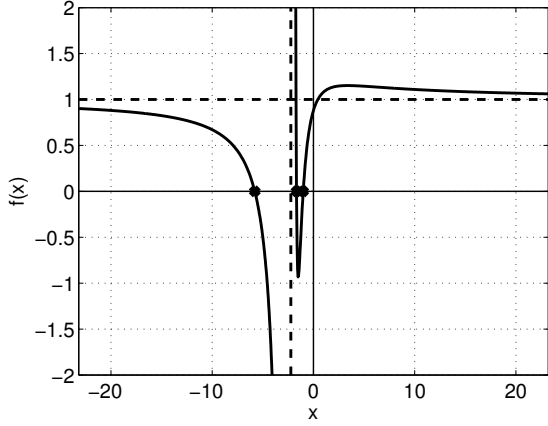


Figure 1: Sample shape of $\Phi(x_k)/C$ defined in (31) with values of parameters given in (33).

Substituting for T in (10) as given in (14) we have,

$$\Phi(x_k) = C \frac{\prod_{i=1}^k (x_k + \beta_i)}{(x_k + \beta)^k}, \quad (31)$$

where $C = (M - k)\varphi(1 - \varphi)^{M-k}$,

$$\beta_i = \begin{cases} L + 1 - (1 - (M - k)\varphi)l_i, & i = 1, \dots, k - 1 \\ \frac{L+1}{(M-k)\varphi}, & i = k \end{cases}, \quad (32)$$

and $\beta = L + 1$. Note that with these definitions we have $0 < \beta_1 < \beta_2 < \dots < \beta_{k-1} < \beta < \beta_k$ and $\sum_{i=1}^k \beta_i - k\beta > 0$. Figure 1 shows a sample shape of $\Phi(x_k)/C$ when $k = 3$ with values of β_i and β defined as,

$$\beta_1 = 1.0082, \beta_2 = 1.6728, \beta_3 = 5.7966, \beta = 2.2310. \quad (33)$$

Note that in this example, it is clear that in any interval in the positive side, the minimum value happens at the boundaries. A lengthy, and yet straight-forward, analysis of (31) proves that this is actually always the case for the β_i and β defined here. We do not go through the mathematics of the proof because of space shortage.

2.5 Proposed Algorithm

- **Aim** Finding the distribution of transmit powers of M mobile stations which gives the maximum aggregate system capacity.

- **Inputs**

1. Number of stations M .
2. Positive values of $g_1 > \dots > g_M$.
3. Background noise I .
4. Minimum quality of service γ .
5. Maximum power of one station p_{max} .
6. Maximum aggregate power P_{max} .

- **Outputs**

1. Transmit power of stations p_1, \dots, p_M .

2. Aggregate relative capacity C .

- **Algorithm**

1. Compute X_{max} as,

$$X_{max} = \frac{P_{max}}{I}.$$

2. Compute φ as,

$$\varphi = \frac{\gamma}{\gamma + 1}.$$

3. For all $1 \leq i \leq M$ compute l_i as,

$$l_i = \frac{p_{max}}{I} g_i.$$

4. For all k satisfying,

$$M - \frac{1}{\varphi} + 1 < k < \frac{1}{\varphi} - \frac{1}{l_M} + 1,$$

do the followings,

- (a) Compute L as,

$$L = \sum_{i=1}^{k-1} l_i.$$

- (b) Compute max_k as,

$$max_k = \min \left\{ l_k, (X_{max} + 1) \left[1 - (M - k)\varphi \right] - L - 1, l_M \frac{1 - (M - k)\varphi}{\varphi} - L - 1 \right\},$$

- (c) Compute min_k as,

$$min_k = \frac{\varphi(L + 1)}{1 - (M - k + 1)\varphi}.$$

- (d) If not $max_k \geq min_k \geq 0$ then go to the next value of k , else continue (goto 4e).

- (e) Do the following lines for two values of $x_k = min_k$ and $x_k = max_k$, separately. Store both ϕ and the values of x_i for each trial.

- Compute T as,

$$1 + T = \frac{x_k + L + 1}{1 - (M - k)\varphi}.$$

- Set $x_i = l_i$ for $i = 1, \dots, k - 1$.

- Set $x_i = \varphi(1 + T)$ for $i = k + 1, \dots, M$.

- Compute ϕ as,

$$\phi = \frac{\prod_{i=1}^M (1 + \sum_{j=1}^M x_j - x_i)}{(1 + \sum_{j=1}^M x_j)^M}.$$

5. Find the smallest ϕ produced at the above and retrieve the corresponding values of x_i .

6. Compute p_i using,

$$p_i = \frac{I x_i}{g_i},$$

and return them.

7. Return C computed as,

$$C = -\log_2 \phi.$$

2.6 Computational Cost

Analysis of the proposed algorithm shows that its computational cost equals $8M^2 + 20M + 10$ flops. As an example, substituting $M = 100$ results in computational costs of 82K flops. In a 1Gfps (one Giga flops per second) processor, it takes less than 0.1ms to do the calculations of the proposed algorithm when M equals 100. As a general measure note that here the computational cost is of order $O(M^2)$.

3. EXPERIMENTAL RESULTS

The proposed algorithm is developed in MATLAB 7.0.4 on a PIV 3.00GHZ with 1GB of RAM. In this platform it takes less than 2ms to give the solution for a cell containing 100 stations.

Note that the parameters of the problem are the sequence g_1, \dots, g_M plus I , γ , p_{max} , and P_{max} . To compare the effects of different parameters, a sample sequence g_i is produced using a set of random numbers that comply with the conditions. To be able to compare the results for different values of M having a controlled variation, we down-sample this sequence to produce new sequences with different lengths (to analyze the effect of number of stations). This way we can compare the effect of selecting different values of different parameters. This analysis is solely for the purpose of showing that the proposed solution is actually functional. Here, we do not intend to give a precise experimental analysis of the problem in hand.

The base parameters used in this study are $\gamma = -30dB$, $I = -113dBm$, $P_{max} = -106dBm$, and $p_{max} = 23dBm$ [2, 1]. Whenever a parameter is different from the list given here it is mentioned. The conversion from dB to a real value is done according to $xdB \equiv 10^{\frac{1}{20}x}$. Also, $xdBm \equiv 10^{\frac{1}{10}x}mw$.

Here, we work on a circular cell of radius $R = 2.5Km$. For the station i at the distance d_i from the base station we only assume the path gain which is calculated as $g_i = Cd_i^n$. Here, C and n are constants equal to 7.75×10^{-3} and -3.66 when d_i is in meters. Equivalently, with d_i in kilometers we will have $C = 1.2283 \times 10^{-13}$ [2]. To produce M values of g_i we uniformly put $3M$ points in $[-R, R] \times [-R, R]$ and from those in the circle with radius R centered at the origin we select M points.

Figure 2 shows the values of the relative aggregate system capacity for different number of stations, M , with varying I and γ . The x -axis in all curves shows the number of stations. Also, each graph shows the effect of choosing between different candidates for I . Clearly, as expected, choosing smaller background noise results in a higher aggregate system capacity. In any situation that the curves stop abruptly, it means that the set of parameters does not yield any result. Again, as expected, by easing the minimum quality of service guaranteed bound, the aggregate system capacity increases.

Similar experiments are carried out to investigate the effects of other parameters, too. Summarizing the effects of different parameters on the relative aggregate capacity, increasing I and γ decreases the total performance. While the first one is a parameter showing the amount of background noise and may be reduced using better engineering, reducing the second one makes the system partial. Also, larger values of p_{max} and P_{max} are better. Though, these values indicate the maximum power which one station affords to transmit and the maximum total interference which is toler-

ated, respectively. These two parameters may not be easily increased. After all, increasing p_{max} with fixed P_{max} may not be beneficial to the aggregate system capacity, because then the condition of total transmission power will push the maximum transmission power of each station down. Also, generally, the system performance decreases as the number of stations increases.

A closer look reveals a negative aspect in the solution to the problem analyzed here. We define the ratio unfairness as the ratio between the maximum and the minimum capacities in the cell and denote it by \tilde{f} . Basically, \tilde{f} is always more than one while smaller values indicate more fair situations. Figure 3 shows the unfairness curves corresponding to the aggregate capacity curves shown in Figure 2. We see that the system is actually very unfair; one station is given a hundred times more bandwidth than another one. This observation is in accordance with what is seen in [8, 4]. Actually, this unfairness was the main reason the authors in [7] devised the minimum signal to noise ratio constraint. Basically, we argue that the impracticality of the solution comes from the fact the definition of the problem neglects the unfairness of the solution. Furthermore, the formulation ignores the fact that while a minimum bound for the signal to noise ratio, and thus for the capacity, is vital there should also be a maximum bound in place too. lack of appropriate constraints on these two important aspects of the problem makes the final results partial and non-realizable. Hence, the future direction of this research is to incorporate more constraints into the problem and to use the interesting mathematical framework introduced here to yield more practical results.

4. CONCLUSIONS

In this paper, the problem of optimizing the aggregate system capacity of reverse link CDMA systems was addressed. In this way, we analyzed a single cell and gave closed form solution to the optimization problem in the cell. Compared to the approach which relies on running a numerical search in some 1-D spaces, the proposed method only demands the computation of the objective function in less than M points in R^M . Sample experiments are conducted and the results are discussed. In addition, in [6, 7] the authors showed that the optimal solution has a semi-bang-bang structure. In the current paper, we establish that it is actually bang-bang.

5. ACKNOWLEDGMENTS

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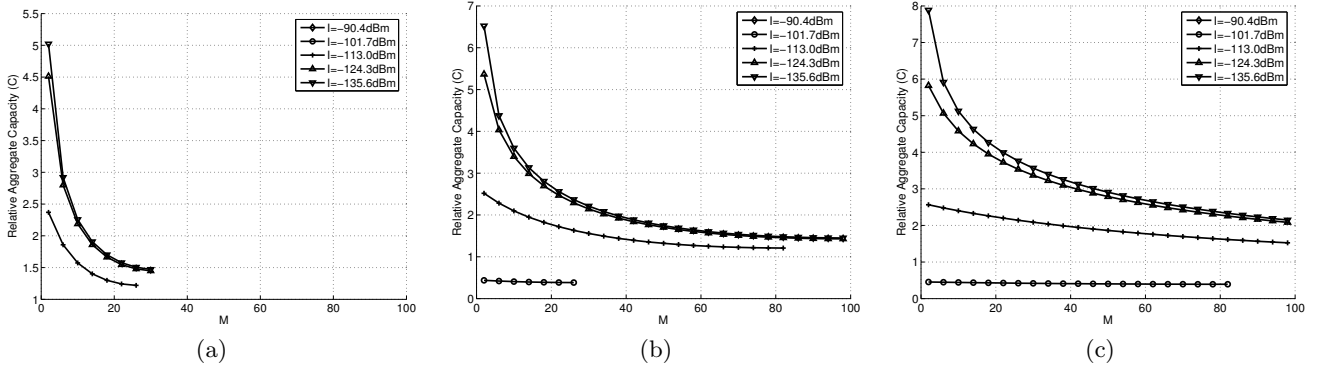


Figure 2: Relative aggregate capacity for different number of stations with varying I and γ . (a) $\gamma = -30\text{dB}$. (b) $\gamma = -40\text{dB}$. (c) $\gamma = -50\text{dB}$.

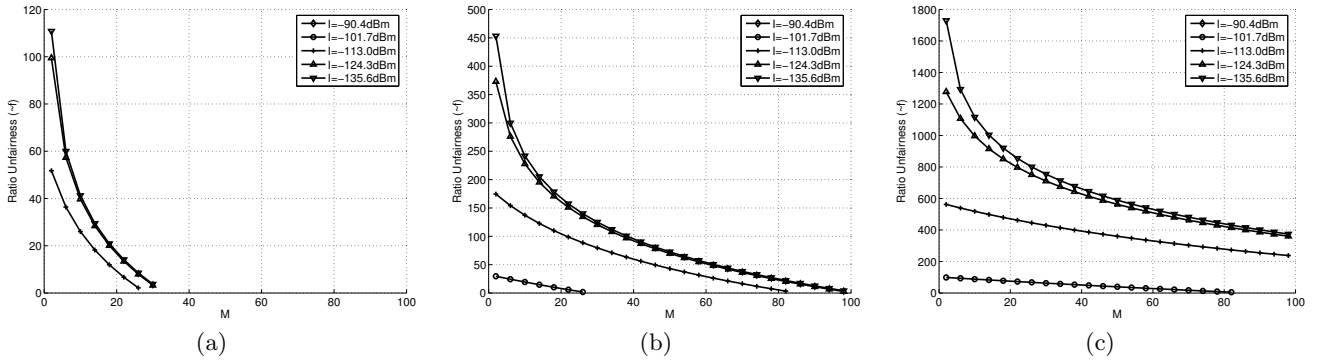


Figure 3: Ratio unfairness for different number of stations with varying I and γ . (a) $\gamma = -30\text{dB}$. (b) $\gamma = -40\text{dB}$. (c) $\gamma = -50\text{dB}$.

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