Information-Theoretic Sum Capacity of Reverse Link CDMA Systems in A Single Cell, An Optimization Perspective

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The information-theoretic approach to maximizing the aggregate capacity of the reverse link in a CDMA system looks for the best pattern of transmission power of the stations. In this framework, where the transmission from each station is noise to all others, extra constraints should be considered to lead to a practically applicable solution. The previous research has suggested a minimum guaranteed quality of service plus bounds on individual transmissions and the aggregate one as constraints. However, extensive analysis has revealed that these two constraints are not enough to produce a solution which can be realized in an actual system. Not imposing any limitation on the maximum capacity of each station and ignoring the unfairness of the whole system is found to be responsible for the partial solution regularly produced by the algorithm. Actually, the classical definition of the problem leads to a solution in which all stations except for one are left to transmit at the lowest possible bandwidth, while the selected station is served with a non-realistic bandwidth of a few hundred times more. Here, we introduce a new constraint to the problem and give an algorithm for solving it. Then, empirical evidence is analyzed to show that the system actually becomes more balanced and practical with the new definition. We show that by imposing a maximum capacity constraint, we can control the share of different station in the aggregate capacity of the system.

Introduction

Assume that we have M mobile stations with the i-th reverse link gain equal to g_i (g_1>...>g_M) and its transmission power equal to $0<p_i<p_{max}$. The SNR for this station equals,

$$\gamma_i = \frac{p_i g_i}{I + \sum_{j=1,j\neq i}^{M} p_j g_j}$$

Approximating the capacity with Shannon formula we have,

$$C_i = \log_2 \frac{I + \sum_{j=1}^{M} p_j g_j}{I + \sum_{j=1,j\neq i}^{M} p_j g_j}$$

For details about the approximation see [2,3]. Now, the problem is to maximize,

$$C(\vec{\mathbf{p}}) = \log_2 \frac{\left(I + \sum_{j=1}^{M} p_j g_j\right)^M}{\prod_{i=1}^{M}\left(I + \sum_{j=1,j\neq i}^{M} p_j g_j\right)}$$

given,

$$0 \leq p_i \leq p_{max}$$

$$\forall i, \gamma_i \geq \gamma$$

$$\sum_{i=1}^{M} p_i g_i \leq P_{max}$$

This problem was first formulated in [1,2] for analyzing single capacities. Then, [3,4,5,6] worked on the aggregate capacity. The formulation discussed here is adopted from [4]. Part of the work introduced here is published in [7].

Here, we add this constraint to the list given in the above,

$$\forall i, C_i \leq \eta$$

Method

Using the transform,

$$x_i = \frac{p_i g_i}{I}$$

We define,

$$l_i = \frac{p_{max}}{I} g_i \qquad \varphi = \frac{\gamma}{\gamma + 1}$$

$$X_{max} = \frac{P_{max}}{I} \qquad \omega = 1 - 2^{-\eta}$$

Now, the problem reduces to minimizing,

$$\Phi(\vec{\mathbf{x}}) = \frac{\prod_{i=1}^{M}\left(1 + \sum_{j=1}^{M} x_j - x_i\right)}{\left(1 + \sum_{j=1}^{M} x_j\right)^M}$$

Given,

$$\forall i, 0 \leq x_i \leq l_i$$

$$\forall i, \varphi \leq \frac{x_i}{1 + \sum_{j=1}^{M} x_j} \leq \omega$$

$$\sum_{i=1}^{M} x_i \leq X_{max}$$

Focusing on the hyperplane,

$$\sum_{i=1}^{M} x_i = T$$

We prove that,

$$\vec{\mathbf{x}} = \Big(\omega(1+T), \cdots, \omega(1+T), l_{j+1}, \cdots, l_{k-1}.x_k, \varphi(1+T), \cdots, \varphi(1+T)\Big)$$

For unknown j and k. Then, an algorithm is given which calculates the optimal j and k, and other variables, in $O(M^3)$. It is worth to mention that the classical problem was solved in $O(M^2)$.

Results

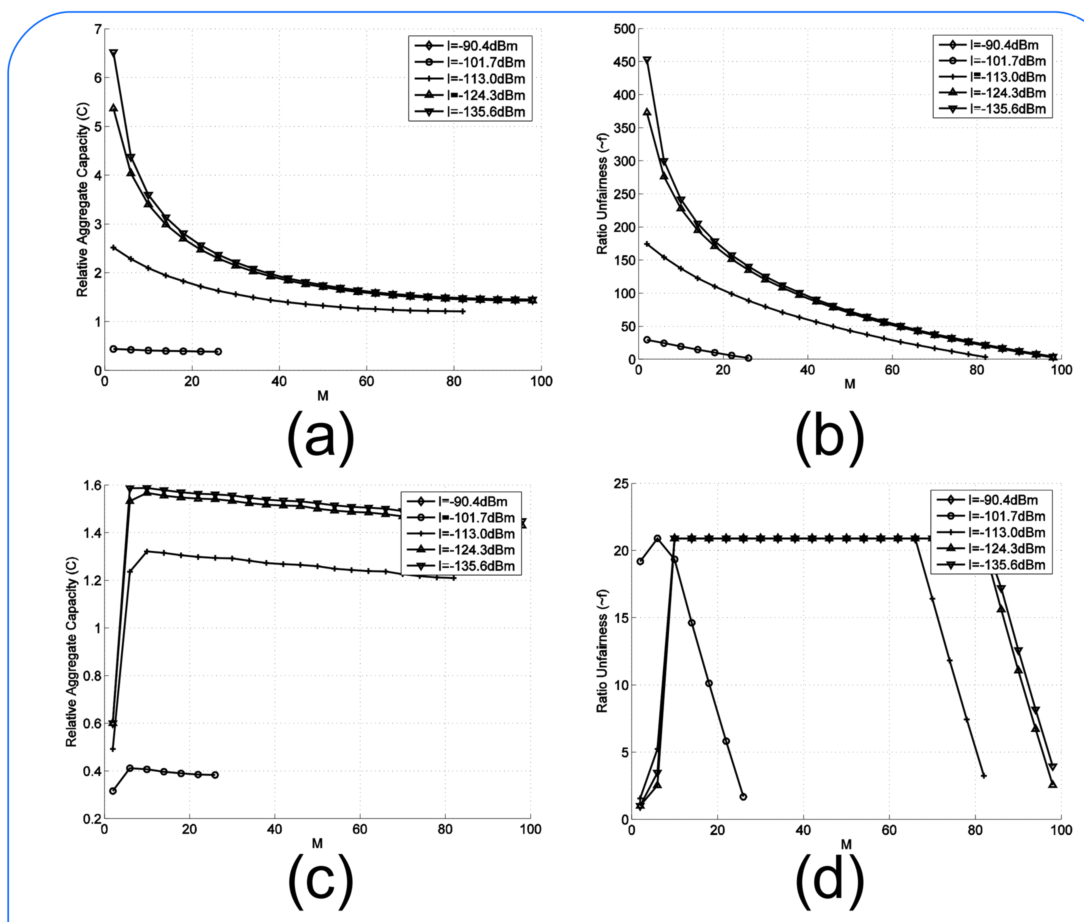

Figure 1, Analyzing the effects of adding the new constraint. (a) and (c) aggregate capacities. (b) and (d) the ratio of the maximum capacity over the minimum one (ratio unfairness measure). (a) and (b) classical problem. (c) and (d) new problem.

As seen in Figure 1, the classical problem is very likely to acquire its revenue by offering a very high bandwidth to the closest station. This can be seen in Figure 1-c where the the ratio unfairness is more than a hundred. In contrary, addition of the maximum constraint limits system's unfairness to great extents (see Figure 1-d).

It is also interesting to see that as the number of stations increases the new problem's aggregate capacity gets closer to the classical one. We emphasize that the aggregate capacity anticipated by the classical problem may not be practically achievable because the “elite” station may not be able to handle the huge bandwidth allocated to it.

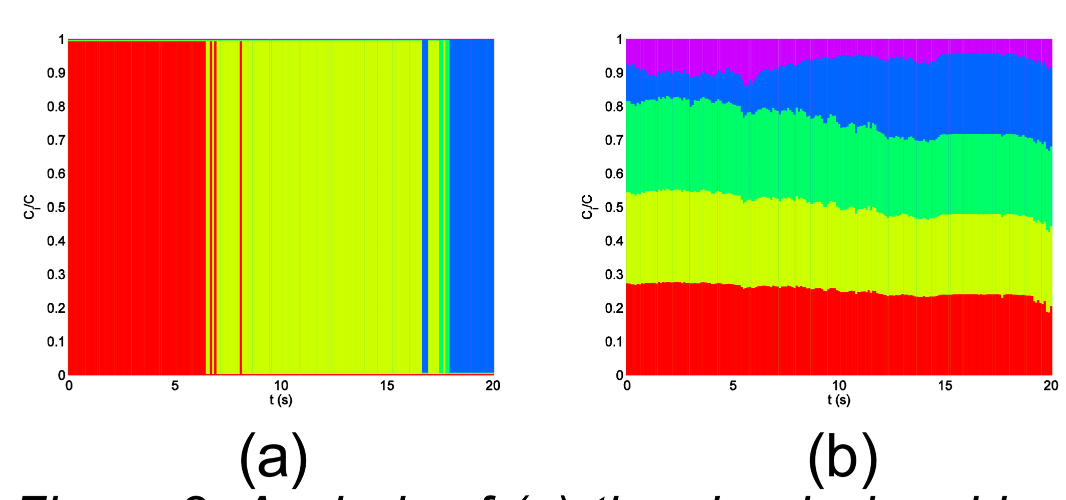

Figure 2, Analysis of (a) the classical problem and (b) new problem during a time period.

Figure 2 caries out the comparison through another scenario. Considering five pedestrian station in a cell, the two problems are solved, as the stations move, and then the capacity shares of different stations over time are visualized. As seen in Figure 2-a, the classical problem tends to allocate all its resources to one station at each moment. In comparison, the new constraints forces the system to maintain a more balanced policy (see Figure 2-b).

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For more graphs and details about the algorithms and the mathematics behind them refer to http://www.win.trlabs.ca/~abadpour/.