Fuzzy Design of A Video–on–Demand Network

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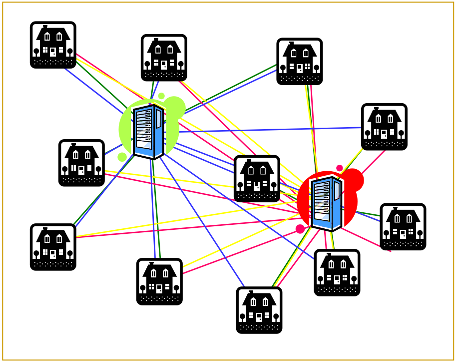
Outline

- Introduction.
- Proposed Method.
- Some Examples, including two scenarios not referred to in the paper.
- Conclusions.

Introduction

- A Video–on–Demand (VoD) system is a dominantly one–way network, connecting a VoD service–provider to customers.
- Looking at the available literature, the common trend is to consider a tree structure for VoD networks, because,
  - Flow in the VoD network is one–directional.
  - Contents of the network, namely video files, are added very gradually and are not ever modified.
- The tree–structured network enables the designer to focus on portions of it and ignore the rest, at different stages, also known as aggregation.

Problem Definition



- The problem is to locate known number of nodes to cache a library of known size in a way which makes the system serve a known population in a near optimal manner.

Problem Modeling

- Weber Problems are used for modeling many location/assignment problems.
- They are known to be hard to solve problems which have multiple solutions.
- The general approach for solving Weber Problems is through a heuristic algorithm called p–median.
  - In data clustering, this algorithm is called Hard C–Means (HCM).
- The zero–one assignment in the classical Weber problem makes it very prone to falling into local minimum.
- After the introduction of Fuzzy Sets, HCM was generalized into Fuzzy C–means (FCM), through the introduction of the concept of fuzziness.

Nomenclature

N	Number of customers.
$\mathbf{X}$	Set of all customers.
$\vec{\mathbf{x}}$	Location of one customer.
$\mu_{\vec{\mathbf{x}}}$	Weight (demand level) of customer $\vec{\mathbf{x}}$.
μ	Total weight of the customers.
l	Library size.
n	Number of nodes.
d_j	Popularity of object j.
$C_{\vec{\mathbf{x}}i}$	Communication cost between customer $\vec{\mathbf{x}}$ and node i.
$\vec{\mathbf{n}}_i$	Location of node i.
l_i	Total resources at node i.
L_{ij}	Allocation for object j at node i.
$p_{\vec{\mathbf{x}}ij}$	Assignment of customer $\vec{\mathbf{x}}$ to node i for object j.

Utilized Models

- Demand: Zipf distribution,

$$d_j = cj^{-\alpha}, \alpha = 0.729.$$

- Communication cost: Power law,

$$C_{\vec{\mathbf{x}}i} = C \left\|\vec{\mathbf{n}}_i - \vec{\mathbf{x}}\right\|^{m_d}.$$

  - The general case is discussed in a companion journal paper,

$$C_{\vec{\mathbf{x}}i} = C\left(\left\|\vec{\mathbf{n}}_i - \vec{\mathbf{x}}\right\|^2\right)$$

  - There is proof of convergence for the cases of $C(x) = x$ and $C(x) = \sqrt{x}$ and we have empirical evidence for $C(x) = x^{\frac{m_d}{2}}, m_d \geq 1$. The general case is an open problem.

Optimization Problem, First Step

We model the expected cost of one transaction.

- Objective Function,

$$\hat{\Delta} = \frac{1}{\mu} \sum_{\vec{\mathbf{x}} \in \mathbf{X}} \left(\mu_{\vec{\mathbf{x}}} \sum_{j=1}^{l} \left(d_j \sum_{i=1}^{n} C_{\vec{\mathbf{x}}i} p_{\vec{\mathbf{x}}ij} \right) \right)$$

- Decision Variables,
  - $p_{\vec{\mathbf{x}}ij}$ and $\vec{\mathbf{n}}_i$.
- Constraint,
  - $\sum_{i=1}^{n} p_{\vec{\mathbf{x}}ij} = 1, \forall \vec{\mathbf{x}}, j$.

What is wrong with this formulation?

- Assignment is binary, thus shortcomings of HCM are anticipated to occur.
- Storage is not addressed.

Optimization Problem, Second Step

Migration from hard to fuzzy.

- Objective Function,

$$\hat{\Delta} = \frac{1}{\mu} \sum_{\vec{\mathbf{x}} \in \mathbf{X}} \left(\mu_{\vec{\mathbf{x}}} \sum_{j=1}^{l} \left(d_j \sum_{i=1}^{n} C_{\vec{\mathbf{x}}i} p_{\vec{\mathbf{x}}ij}^{m} \right) \right)$$

- Decision Variables,
  - $p_{\vec{\mathbf{x}}ij}$ and $\vec{\mathbf{n}}_i$.
- Constraint,
  - $\sum_{i=1}^{n} p_{\vec{\mathbf{x}}ij} = 1, \forall \vec{\mathbf{x}}, j$.

What is wrong with this formulation?

- Assignment is binary, thus problems HCM are anticipated to occur.
- Storage is not addressed.

Implicit Inclusion of Storage Cost

- We are trying to produce a one–expression goal function which includes both communication cost and storage cost.
- A linear combination of the two costs is not what we are looking for,
  - Because, we want our objective function to resemble that of FCM.
- Therefore, we use the term φ_{ij}

$$\varphi_{ij} = 1 + \left(\frac{d_j L_{ij}}{L}\right)^{-k}$$

We will discuss what this term does shortly.

Optimization Problem

- Objective Function,

$$\hat{\Delta} = \frac{1}{\mu} \sum_{\vec{\mathbf{x}} \in \mathbf{X}} \left(\mu_{\vec{\mathbf{x}}} \sum_{j=1}^{l} \left(d_j \sum_{i=1}^{n} C_{\vec{\mathbf{x}}i} p_{\vec{\mathbf{x}}ij}^{m} \varphi_{ij} \right) \right)$$

- Decision Variables $p_{\vec{\mathbf{x}}ij}$, L_{ij} and $\vec{\mathbf{n}}_i$.
- Mediator Variable: $\varphi_{ij} = 1 + \left(\frac{d_j L_{ij}}{L} \right)^{-k}$.
  - We will talk about φ_{ij} shortly.
- Constraints,
  - $\sum_{i=1}^{n} L_{ij} = \mu, \forall j$.
  - $\sum_{i=1}^{n} p_{\vec{\mathbf{x}}ij} = 1, \forall \vec{\mathbf{x}}, j$.

The Role of φ_{ij}

- We know that,
  - As L_{ij} tends to zero, φ_{ij} approaches infinity and for $d_j L_{ij} \geq L$, φ_{ij} tends to one.
- This term carries out three goals,
  - Helps satisfy $L_{ij} \simeq D_{ij}, \forall i, j$
  - Forces small $d_j L_{ij}$s to become zero.
  - Becomes “transparent” when the solution converges.

Hint:

$$\varphi_{ij} = 1 + \left(\frac{d_j L_{ij}}{L}\right)^{-k}$$

$$\hat{\Delta} = \frac{1}{\mu} \sum_{\vec{\mathbf{x}} \in \mathbf{X}} \left(\mu_{\vec{\mathbf{x}}} \sum_{j=1}^{l} \left(d_j \sum_{i=1}^{n} C_{\vec{\mathbf{x}} i} p_{\vec{\mathbf{x}} ij}^{m} \varphi_{ij}\right)\right)$$

Summary: Optimization Problem

- Minimize,

$$\hat{\Delta} = \frac{1}{\mu} \sum_{\vec{\mathbf{x}} \in \mathbf{X}} \left(\mu_{\vec{\mathbf{x}}} \sum_{j=1}^{l} \left(d_j \sum_{i=1}^{n} C_{\vec{\mathbf{x}}i} p_{\vec{\mathbf{x}}ij}^{m} \varphi_{ij} \right) \right)$$

- Subject to,
  - $\sum_{i=1}^{n} L_{ij} = \mu, \forall j.$
  - $\sum_{i=1}^{n} p_{\vec{\mathbf{x}}ij} = 1, \forall \vec{\mathbf{x}}, j.$
- Here, the decision variables are $p_{\vec{\mathbf{x}}ij}$, L_{ij} and $\vec{\mathbf{n}}_i$.

Proposed Solver

- Randomly generate a network which complies with all the constraints.
-
  - Given the known caching and the known location of the nodes generate the best assignment.
  - Given the known assignment and the known location of the nodes generate the best caching.
  - Given the known caching and the known assignment of the nodes generate the best location of the nodes.
- Until convergence.

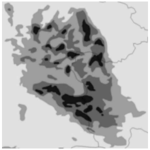
For details refer to the paper.

Application Scenarios

- Design the whole network (a more theoretical application).
- Locate one node after the population changes.
- Recalculate the caching strategy.

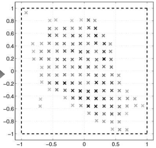
The last two are not mentioned in the paper.

How the Algorithm Works: A Hypothetical Problem



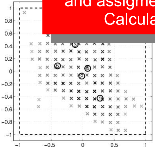
Population Density

Aggregation



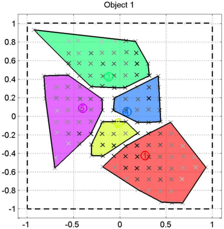
Aggregated Customers

Developed Method

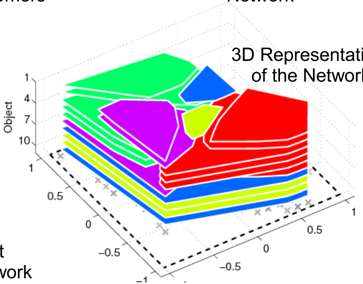


Nodes, caching strateg and assigment are all Calculated

Calculated Network



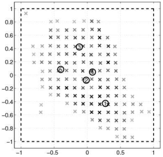
Distribution of the Most Popular Object in the Network



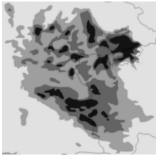
3D Representation of the Network

Calculating an optimal network for a given population density.

How the Algorithm Works: Network Growth



Available Network

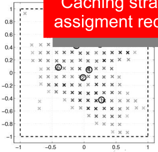


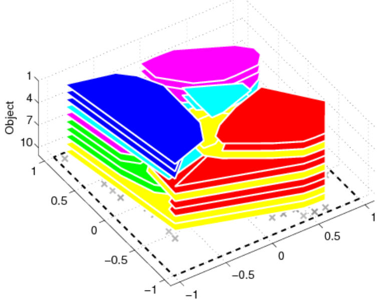
Population Growth

Developed Method

A new node added.

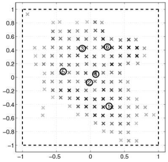
Caching strategy and assigment recalculated



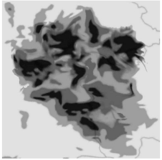


Dealing with partial growth.

How the Algorithm Works: Change in Demand



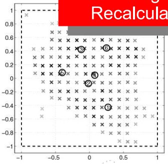
Available Network

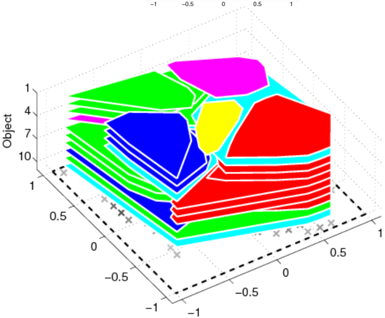


Change in Demand

Developed Method

Caching strategy and assigment Recalculated.





Dealing with change in demand.

Conclusions

- An iterative fuzzy method is developed to design and maintain a video–on–demand network.
- The algorithm addresses communication in the network as well as the storage.
- Through dynamic reshaping of the algorithm, the same code can be utilized for different purposes, including the hypothetical design of the whole network, its staged growth, and the maintenance of caching and assignment in it.

Thank you for your time...

Any Questions?