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ScienceDirect
Int. J. Electron. Commun. (AEÜ) 63 (2009) 108–115
International Journal AEÜ of Electronics and Communications
www.elsevier.de/aeue

Approximation algorithms for maximizing the information – theoretic sum capacity of reverse link CDMA systems

Arash Abadpour[a,b,*], Attahiru Sule Alfa[a,b], Anthony C.K. Soong[c]

[a] *Department of Electrical and Computer Engineering, University of Manitoba, Canada*
[b] *TRLabs, Winnipeg, Canada*
[c] *Huawei Technological Co. Ltd., TX, USA*

Received 14 July 2007; accepted 6 November 2007

Abstract

Since the introduction of multimedia services to CDMA systems, many researchers have been working on the maximization of the aggregate capacity of the reverse link. This problem looks for the optimal set of transmission powers of the stations subject to a set of constraints. One of the research directions in this field is to devise a practically realistic set of constraints and then to propose an algorithm for solving the resulting problem. Through a unified approach, introduced recently by the authors, a more general investigation of the problem, equipped with a wide range of constraints, is possible. Here, we go further and propose an approximation to reshape the objective function into a more conveniently workable one. Then, we analyze the three available formulations of the problem and show that integrating this approximation into the available algorithms has the benefit of reducing the computational cost. The paper includes the mathematics involved in the approximation and its integration into the algorithms. Also, we analyze examples to demonstrate the achievements of the proposed method.

Keywords: Quality of service; Single cell; Reverse link CDMA; Optimization; Approximation

1. Introduction

Analysis of the information-theoretic capacity of the reverse link in a CDMA system was first addressed in [1]. That work, plus other earlier ideas for multi-user networks [2,3], are surveyed in [4]. These works focus on the set of all capacities based on multi-user detection techniques. More recent works analyze the aggregate reverse link capacity, using matched filters [5–7].

A challenge in modeling these systems is to suggest a proper mapping between the signal to noise ratio (SNR) and the throughput. This mapping would be determined by the coding [7], when a sub-optimal coding strategy is used. Nevertheless, the assumption of Shannon capacity is theoretically feasible because of the existence of coding strategies such as Turbo Coding [8].

Here, we first introduce the formal formulation of the problem. Then, different approaches to the problem will be discussed. Assume that M stations are given in a cell, where the ith station is transmitting at power $p_i \in [0, p_{\max}]$. The path gain from this station to the base equals g_i and we assume that $g_1 \geqslant \cdots \geqslant g_M$. Now, the ith station's SNR is modeled as

$$\gamma_i = \frac{p_i g_i}{I + \sum_{j=1, j\neq i}^{M} p_j g_j}. \tag{1}$$

Note that, here, we are looking at the chip level and thus the signal to interference ratio (SIR) is equal to the SNR.

* Corresponding author at: Department of Electrical and Computer Engineering, University of Manitoba, Canada.
E-mail addresses: abadpour@ee.umanitoba.ca, abadpour@win.trlabs.ca (A. Abadpour), alfa@ee.umanitoba.ca, alfa@win.trlabs.ca (A.S. Alfa), asoong@huawei.com (A.C.K. Soong).

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doi:10.1016/j.aeue.2007.11.002

Using the Shannon theorem, the relative capacity of the ith station is well approximated as [4],

$$C_i = \log_2(1+\gamma_i). \tag{2}$$

This capacity is called relative, because we have ignored the fixed transmission bandwidth, B. Now, the problem is to maximize,

$$C = \sum_{i=1}^{M} C_i. \tag{3}$$

To make the system practically applicable, the aggregate power received by the base station must be below a preselected value,

$$\sum_{i=1}^{M} p_i g_i \leqslant P_{\max}. \tag{4}$$

Also, it is necessary to have a minimum guaranteed bound on the SNR of the single stations, $\gamma_i \geqslant \gamma$. This formulation is adopted from [8] and will be called the classical single-cell (CSC) problem.

Oh and Soong [8] worked on CSC and showed that rather than searching for the solution in $[0, p_{\max}]^M$, the search can be conducted in a set of M one-dimensional sub-spaces. For that, they utilized a numerical search. This problem was revisited in [9] using a more general framework. There, the authors proved that the actual search space can be further limited to a set of less than $2M$ points, for which closed forms were given. The proposed algorithm was then shown to demand $O(M^2)$ flops, more than nine times less than the previous approach.

Analysis of CSC reveals that almost always there is one station in the cell which transmits at a capacity about 50 times as mush as the others [6,7,10]. To deal with this unfair behavior, and using the mathematical approach introduced in [9], a maximum capacity constraint, as $C_i \leqslant \eta$, is added to the problem, resulting in the new single-cell (NSC) problem [10]. The algorithm proposed in [10] solves the NSC for a M-station cell in $O(M^3)$ flops. The essential structure of the NSC algorithm is similar to that of the CSC, except for the fact that the definition of the intervals and the structure of the solution are different in the two cases.

In order to enhance the results of NSC, a new constraint was added to the problem, resulting in N$^+$SC [11]. This was carried out by defining the capacity share of the ith station as $\tilde{C}_i = C_i/C$ and then adding the inequality $\tilde{C}_i \leqslant (M\mu)^{-1}$ to the constraints. It was shown that the solution to N$^+$SC either comes from a NSC or from a different version of it, called N′SC. In either case, the algorithm works in known boundaries and checks at most $M(M-1)$ possible solutions. N$^+$SC, which demands $O(M^3)$ flops, uses an approximation for $\tilde{C}_i$.

In this paper, we propose a new set of approximations to simplify the three problems. This simplification results in simpler proofs for the theorems presented in [9–11]. Also, we show that these approximations enable us to reduce the computational cost of each algorithm by one in powers of M. Using the superscript a for the new algorithms, we propose algorithms CSC^a, NSC^a, and N$^+$SC^a, for solving the problems CSC, NSC, and N$^+$SC in $O(M)$, $O(M^2)$, and $O(M^2)$, respectively.

2. Proposed method

2.1. Mathematical method

Using the linear transformation,

$$x_i = \frac{p_i g_i}{I} \tag{5}$$

the goal function reduces to maximizing,

$$C(\vec{\mathbf{x}}) = \frac{(1+\sum_{j=1}^{M} x_j)^M}{\prod_{i=1}^{M}(1+\sum_{j=1,j\neq i}^{M} x_j)}. \tag{6}$$

This relationship can be derived by substituting (5) in (1) and the using (2). Similarly, the constraints can be written as,

$$x_i \in [0, l_i], \quad l_i = \frac{p_{\max} g_i}{I}, \tag{7}$$

$$1-2^{-\eta} = \omega \geqslant \frac{x_i}{1+\sum_{j=1}^{M} x_j} \geqslant \varphi = \frac{\gamma}{\gamma+1}, \quad \forall i, \tag{8}$$

$$\sum_{i=1}^{M} x_i \leqslant X_{\max} = \frac{P_{\max}}{I}. \tag{9}$$

Here, setting $\omega = \infty$ and $\mu = 0$ gives CSC. Also, setting $\mu = 0$ gives NSC and no restriction leads to N$^+$SC.

It is shown that investigating the behavior of the problem in the set of hyperplanes defined as $\sum_{i=1}^{M} x_i = T$ is beneficiary [9]. In these hyperplanes, the bound for the aggregate transmission power changes into $T \leqslant X_{\max}$. Also, the two other constraints add up into,

$$x_i \in [\varphi(1+T), \min\{l_i, \omega(1+T)\}]. \tag{10}$$

It is proved that [9], if we can limit the search space to $x_i \in [b_i, B_i]$, where b and B_i's are positive values and B_i's are sorted in a descending fashion, then the maximum of $C(\vec{\mathbf{x}})$ occurs when $\vec{\mathbf{x}} = (B_1, \ldots, B_{k-1}, x_k, b, \ldots, b)$ for values of k and x_k yet to be found. This property is called the boundary theorem and is a result of another theorem which states that the distance between x_i's should be as large as possible.

2.2. Typical algorithm

The three algorithms of CSC, NSC, N$^+$SC, and also the internal algorithm N′SC, have a fairly similar structure. Independent analysis of each problem shows that there is a

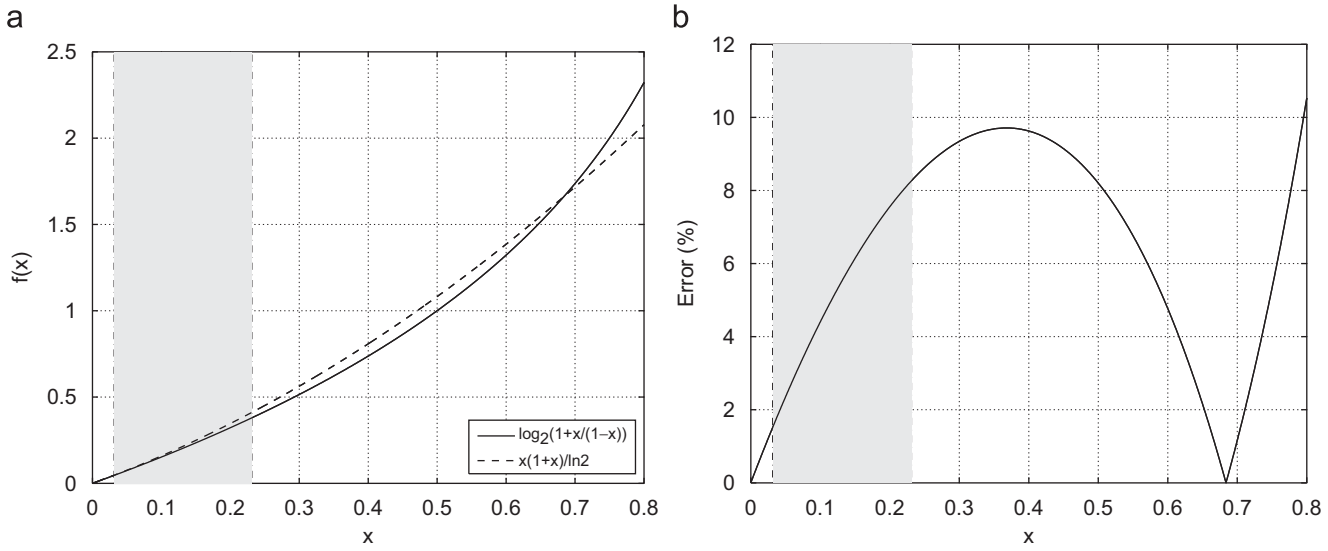

Fig. 1. Investigating the properness of the approximation given in (12): (a) the two sides; and (b) relative error. NSC and N$^+$SC work in the shaded area.

value of k, or a pair of values j and k, for which the vector $\vec{\mathbf{x}}$ is linearly calculated based on x_k. Then, the problem is to find the best k, or k and j, and then spot the best x_k. For each problem, it is independently proved that having fixed k, and also j if applicable, x_k should accept either the smallest or the largest value mandated by the boundaries. This way, in each algorithm, the two functions θ and Θ are derived, both of which depend on k, and j if applicable, and system parameters. Then, the task is to iterate over all values of k, and j if applicable, and to set $x_k = \theta$ and $x_k = \Theta$ and gather all possible results. Then, the best solution is selected.

2.3. Approximation

Except for CSC, we have $\gamma_i \in [\gamma, 2^\eta - 1]$. To reach to an appropriate approximate form, we note that for nominal values of $\gamma = -30$ dB and $\eta = 0.3$, approximating $\ln(1+\gamma_i)$ with γ_i carries less than 10% error. Hence, we write,

$$C_i \simeq \frac{1}{\ln 2}\gamma_i = \frac{1}{\ln 2}\left[\frac{x_i/(1+T)}{1-x_i/(1+T)}\right] \simeq \frac{1}{\ln 2}\left[\frac{x_i}{1+T}\left(1+\frac{x_i}{1+T}\right)\right]. \quad (11)$$

Here, the approximation,

$$\log_2\left(1+\frac{x}{1-x}\right) \simeq \frac{1}{\ln 2}x(1+x). \quad (12)$$

Fig 1a shows that for the cases of NSC and N$^+$SC, the shaded area is being used, the approximation presented in (12), results in less than 8% error. Also, for these two problems, the actual value is always less than the approximated value. However, for highly impartial CSC solutions, for which x may go over 0.7, the approximation may fall below the actual value. Nevertheless, the error is always less than 10%.

Using (11) we have,

$$C(\vec{\mathbf{x}}) \simeq \frac{1}{\ln 2}\left(1+\frac{\sum_{i=1}^{M}x_i^2}{(1+T)^2}-\frac{1}{1+T}\right), \quad (13)$$

which shows that, for the fixed T, the maximum value of $C(\vec{\mathbf{x}})$ happens when $\sum_{i=1}^{M}x_i^2$ is maximum. Assuming that all x_i's are fixed, except for x_j and x_k, and that $x_j + x_k = T - \sum_{i=1, i\notin\{j,k\}}^{M} x_i = S$, the search reduces to maximizing $x_j^2 + x_k^2$. Because of $2(x_j^2 + x_k^2) = (x_j + x_k)^2 + (x_j - x_k)^2$, the maximum happens when $|x_j - x_k|$ is maximum. This is identical to the theorem proved in [9].

Note that the approximation given in (13) is also helpful in faster calculation of $C(\vec{\mathbf{x}})$. As described in Section 2.2, the typical algorithm includes finding bounds for x_k for different values of k, and j if applicable, and then finding the aggregate relative capacity at those bounds. Using the approximation given here, the calculations can be performed faster. Hence, if the approximate algorithm does not fail in finding the best k, and j if applicable, the values of $p_1, \ldots, p_M$ would be exact.

The actual worry here is that the approximation may deviate the algorithm from finding the best k, or j, and hence may produce a wrong result. Here, we assume that the approximation does not change two values of C for two different sets of p_i's in a way that the better solution becomes worse. Through experimental analysis it will be empirically shown that the approximate algorithm does give the exact solution, with a negligible probability of erratic behavior.

Note that, after p_i's are found, C will be calculated again, using the exact calculation, in order to yield the exact result.

2.4. Approximate algorithms

Lemma I. *Assume that for given values of a, b and $c > 0$, the function,*

$$f(x) = \frac{x^2 + ax + b}{(x + c)^2}, \tag{14}$$

is given. For any interval on the positive side, say $[\theta, \Theta]$, the maximum value of $f(x)$ in $[\theta, \Theta]$ is either $f(\theta)$ or $f(\Theta)$.

Proof. The lemma can be proved by analyzing the behavior of $f(x)$ and $f'(x)$ at the boundaries. □

Using this lemma, the three approximate algorithms are presented below.

2.4.1. CSC^a

According to the CSC results, we have [9],

$$\vec{\mathbf{x}} = (l_1, \ldots, l_{k-1}, x_k, \varphi(1+T), \ldots, \varphi(1+T)), \tag{15}$$

resulting in,

$$C(\vec{\mathbf{x}}) \simeq \frac{1}{\ln 2}\left(\psi^2 \frac{x_k^2 - (1/\psi)x_k + L - (1/\psi)(l+1)}{(x_k + l + 1)^2} + 1 + (M-k)\varphi^2\right). \tag{16}$$

Here,

$$L = \sum_{i=1}^{k-1} l_i^2, \tag{17}$$

$$T = \frac{x_k + l + 1}{\psi}, \tag{18}$$

$$l = \sum_{i=1}^{k-1} l_i, \tag{19}$$

$$\psi = 1 - (M-k)\varphi. \tag{20}$$

Now, the problem of finding the best x_k translates into finding the maximum of $f(x)$, as defined in Lemma I, where,

$$a = -\frac{1}{\psi}, \quad b = L - \frac{l+1}{\psi}, \quad c = l + 1. \tag{21}$$

Here, the search is performed for all x_k's which satisfy [9],

$$x_k \leqslant \min\left\{ l_k, \psi \min\left\{ \begin{matrix} X_{\max} + 1 \\ \frac{1}{\varphi} l_M \end{matrix} \right\} - (l+1) \right\}, \tag{22}$$

$$x_k \geqslant \frac{\varphi}{\psi - \varphi}(l+1). \tag{23}$$

Hence, using Lemma I, x_k must accept one of the bounds given in (22) and (23). Using this method, the computational cost of CSC^a becomes $36M + 6$ flops, compared to the $8M^2 + 20M + 10$-flop cost of CSC [9].

2.4.2. NSC^a

Analysis of NSC shows that [10],

$$\vec{\mathbf{x}} = (\omega(1+T), \ldots, \omega(1+T), l_{j+1}, \ldots, l_{k-1}x_k, \varphi(1+T), \ldots, \varphi(1+T)), \tag{24}$$

for which,

$$C(\vec{\mathbf{x}}) \simeq \frac{1}{\ln 2}\left(\psi^2 \frac{x_k^2 - (1/\psi)x_k + L - (1/\psi)(l+1)}{(x_k + l + 1)^2} + 1 + (M-k)\varphi^2 + j\omega^2\right). \tag{25}$$

Here,

$$L = \sum_{i=j+1}^{k-1} l_i^2, \tag{26}$$

$$l = \sum_{i=j+1}^{k-1} l_i, \tag{27}$$

$$T = \frac{x_k + l + 1}{\psi}, \tag{28}$$

$$\psi = 1 - (j\omega + (M-k)\varphi). \tag{29}$$

Now, using Lemma I, x_k should accept one of the limits of [10],

$$x_k \leqslant \min\left\{ \begin{matrix} l_k, \\ \frac{\omega}{\psi - \omega}(L+1), \psi > \omega \\ \psi \min\left\{ \begin{matrix} \frac{1}{\omega} l_j, j \neq 0 \\ \frac{1}{\varphi} l_M \\ X_{\max} + 1 \end{matrix} \right\} - (L+1) \end{matrix} \right\}, \tag{30}$$

$$x_k \geqslant \max\left\{ \begin{matrix} \frac{\psi}{\omega} l_{j+1} - (L+1), k > j+1 \\ \frac{\varphi}{\psi - \varphi}(L+1) \end{matrix} \right\}. \tag{31}$$

Here, $[P]$ is one if the condition P holds and zero otherwise. Using the approximate closed form for $C(\vec{\mathbf{x}})$ given in (25), the computational cost of NSC^a reduces to $23M^2 - 21M + 14$ flops, compared to the $\frac{16}{3}M^3 + \frac{55}{3}M^2 - 23M + 6$-flop cost of NSC [10].

2.4.3. N^+SC^a

To analyze N^+SC^a, we first have to analyze $N'SC^a$. In $N'SC$ we have [11],

$$\vec{\mathbf{x}} = \left(\frac{1}{M\mu}T, \ldots, \frac{1}{M\mu}T, l_{j+1}, \ldots, l_{k-1}, x_k, \varphi(1+T), \ldots, \varphi(1+T)\right), \quad (32)$$

which gives,

$$C(\vec{\mathbf{x}}) \simeq \frac{1}{\ln 2}\left(\frac{\psi^2}{(x_k+l+\alpha)^2}\left(x_k^2 - \frac{1}{\psi}\left(\frac{2j}{M^2\mu^2}+1\right)x_k + L + \frac{j}{M^2\mu^2} - \frac{1}{\psi}\left(\frac{2j}{M^2\mu^2}+1\right)(l+\alpha)\right) + 1 + (M-k)\varphi^2 + \frac{j}{M^2\mu^2}\right). \quad (33)$$

Here,

$$L = \sum_{i=j+1}^{k-1} l_i^2, \quad (34)$$

$$l = \sum_{i=j+1}^{k-1} l_i, \quad (35)$$

$$1 + T = \frac{x_k + l + \alpha}{\psi}, \quad (36)$$

$$\alpha = 1 - \frac{j}{M\mu}, \psi = 1 - \left(\frac{j}{M\mu} + (M-k)\varphi\right). \quad (37)$$

Now, using Lemma I, the solution is one of the boundaries given as [11],

$$x_k \leqslant \min\left\{\begin{array}{l} \psi \min\left\{\begin{array}{l} X_{\max}+1 \\ M\mu l_j + 1,\ j \neq 0 \\ \dfrac{1}{\varphi} l_M \end{array}\right\} - (L+\alpha) \\ \dfrac{L+\alpha-\psi}{M\mu\psi - 1},\ M\mu\psi > 1 \\ l_k \end{array}\right\}, \quad (38)$$

$$x_k \geqslant \max\left\{\begin{array}{l} \psi(M\mu l_{j+1}+1) - (L+\alpha),\ k > j+1, \\ \dfrac{\varphi}{\psi-\varphi}(L+\alpha) \end{array}\right\}. \quad (39)$$

Using (33), the computational cost of $N'SC^a$ reduces from $\frac{16}{3}M^3 + \frac{67}{3}M^2 - 27M + 16$ flops to $26M^2 - 23M + 15$ flops.

Using these results, and also the ones presented in Section 2.4.2, the computational cost of N^+SC^a becomes $51M^2 - 46M + 33$ flops. This figure should be compared to the cost of N^+SC which is $\frac{32}{3}M^3 + \frac{128}{3}M^2 - 52M + 20$ flops [11].

3. Experimental results

The proposed algorithms are developed in MATLAB 7.0.4 on a PIV 3.00 GHZ personal computer with 1 GB of RAM. The parameters used in this study are, $\gamma = -40$ dB, $I = -113$ dB m, $P_{\max} = -106$ dB m, $p_{\max} = 23$ dB m, $\eta = 0.3$,

Table 1. Investigating the properness of the approximation

Station #		1	2	3	4	5	6	7
$g_i(\times 10^{-11})$		0.11	0.031	0.0067	0.0018	0.0011	0.00069	0.00052
CSC	p_i	21.13	0.96	4.45	16.51	25.96	43.20	57.23
	γ_i	3.430	0.010	0.010	0.010	0.010	0.010	0.010
	$\hat{p}_i$	0.774	0.010	0.010	0.010	0.010	0.010	0.010
	C_i	2.147	0.014	0.014	0.014	0.014	0.014	0.014
	C_i^a	**1.98** (8%)	**0.01** (0%)	**0.01** (0%)	**0.01** (0%)	**0.01** (0%)	**0.01** (0%)	**0.01** (0%)
		$C = 2.233$, $C^a = \mathbf{2.068}$(7%)						
NSC	p_i	5.12	18.11	84.46	199.53	199.53	199.53	167.38
	γ_i	0.231	0.231	0.231	0.136	0.082	0.048	0.030
	$\hat{p}_i$	0.188	0.188	0.188	0.120	0.076	0.046	0.029
	C_i	0.300	0.300	0.300	0.184	0.114	0.068	0.042
	C_i^a	**0.32** (7%)	**0.322**(7%)	**0.32** (7%)	**0.19** (5%)	**0.12** (3%)	**0.07** (2%)	**0.04** (1%)
		$C = 1.308$, $C^a = \mathbf{1.389}$ (6%)						
N^+SC	p_i	4.52	15.98	74.50	199.53	199.53	199.53	199.53
	γ_i	0.214	0.214	0.214	0.146	0.088	0.051	0.038
	$\hat{p}_i$	0.176	0.176	0.176	0.127	0.081	0.049	0.037
	C_i	0.280	0.280	0.280	0.197	0.122	0.072	0.054
	C_i^a	**0.299** (7%)	**0.299** (7%)	**0.299** (7%)	**0.207** (5%)	**0.126** (4%)	**0.074** (2%)	**0.055** (2%)
		$C = 1.284$, $C^a = \mathbf{1.369}$ (6%)						

The values in parentheses show relative error. Italic and bold text shows the final results of the exact and approximate algorithms, respectively.

Table 2. Investigating the properness of the approximation when embedded into the algorithms

Station #			1	2	3	4	5	6	7
g_i ($\times 10^{-11}$)			0.4	0.0051	0.0038	0.0019	0.0014	0.0008	0.00052
CSC^a	E	p_i	*5.77*	*5.90*	*7.85*	*16.02*	*21.28*	*37.19*	*57.24*
		C_i	*2.147*	*0.014*	*0.014*	*0.014*	*0.014*	*0.014*	*0.014*
		C				*2.233*			
	A	p_i^a	5.77 (0%)	5.90 (0%)	7.85 (0%)	16.02 (0%)	21.28 (0%)	37.19 (0%)	57.24 (0%)
		C_i^a	1.982 (8%)	0.014 (0%)	0.014 (0%)	0.014 (0%)	0.014 (0%)	0.014 (0%)	0.014 (0%)
		C^a				2.068 (8%)			
	A + E	$C_i^{a\star}$	**2.147** (**0**%)	**0.014** (**0**%)	**0.014** (**0**%)	**0.014** (**0**%)	**0.014** (**0**%)	**0.014** (**0**%)	**0.014** (**0**%)
		$C^{a\star}$				**2.233** (**0**%)			
NSC^a	E	p_i	*1.40*	*111.86*	*148.95*	*199.53*	*199.53*	*166.60*	*57.24*
		C_i	*0.300*	*0.300*	*0.300*	*0.190*	*0.141*	*0.065*	*0.014*
		C				*1.310*			
	A	p_i^a	1.40 (0%)	111.86 (0%)	148.95 (0%)	199.53 (0%)	199.53 (0%)	166.60 (0%)	57.24 (0%)
		C_i^a	0.322 (7%)	0.322 (7%)	0.322 (7%)	0.200 (5%)	0.146 (4%)	0.067 (2%)	0.014 (0%)
		C^a				1.393 (6%)			
	A + E	$C_i^{a\star}$	**0.300** (**0**%)	**0.300** (**0**%)	**0.300** (**0**%)	**0.190** (**0**%)	**0.141** (**0**%)	**0.065** (**0**%)	**0.014** (**0**%)
		$C^{a\star}$				**1.310** (**0**%)			
N$^-$SC^a	E	p_i	*1.33*	*106.43*	*141.72*	*199.53*	*199.53*	*199.53*	*164.49*
		C_i	*0.284*	*0.284*	*0.284*	*0.190*	*0.141*	*0.079*	*0.042*
		C				*1.303*			
	A	p_i^a	1.33 (0%)	106.43 (0%)	141.72 (0%)	199.53 (0%)	199.53 (0%)	199.53 (0%)	164.49 (0%)
		C_i^a	0.304(7%)	0.304 (7%)	0.304 (7%)	0.200 (5%)	0.146 (4%)	0.081 (2%)	0.042 (1%)
		C^a				1.303 (0%)			
	A + E	$C_i^{a\star}$	**0.284** (**0**%)	**0.284** (**0**%)	**0.284** (**0**%)	**0.190** (**0**%)	**0.141** (**0**%)	**0.079** (**0**%)	**0.042** (**0**%)
		$C^{a\star}$				**1.303** (**0**%)			

The values in parentheses show relative error. Italic and bold text shows the final results of the exact and approximate algorithms, respectively.

$\mu = \frac{1}{1.5}$, and $M = 7$ [12]. Here, we work on a circular cell of radius $R = 2.5$ km. For the station i, at distance d_i from the base station, we only consider the path-loss, which is calculated as $g_i = cd_i^n$ [13]. Here, $c = 7.75 \times 10^{-3}$ and $n = -3.66$ and d_i is in meters [12]. In the following, the superscript a is used for all variables which are calculated using the approximate forms.

Table 1 investigates the properness of the applied approximation for a system solved in the three frameworks. Note that, as expected, the error in C_i^a is always less than 10% in all cases. The least error is observed in CSC for those stations which transmit at the minimum capacity. This was predicatble, because based on Fig. 1b, we know that smaller values of $x_i/(1+T)$ undergo smaller errors. Also, note that, as again expected from the discussion given for Eq. (12), the approximation is always producing excessive results for NSC and N$^+$SC. Furthermore, as again expected, the approximation is sometimes conservative in the case of CSC, but not always.

Table 2 shows the results of solving a sample problem with the three algorithms. For each algorithm, first the exact results are shown (E). Then, the raw results of the approximate algorithm are presented (A), followed by the results of finding the exact values using the approximate solutions (A + E). These last ones are the final outputs of the CSC^a, NSC^a, and N$^+$SC^a algorithms. As stated at the end of Section 2.3, as the proposed algorithms find precise values of the boundaries, the value of p_i^a is always identical to the values of p_i. However, There exists a chance that the approximation may deviate the optimization process from finding the real maximum, due to the induced errors. In this experiment we did not observe such an event.

In an effort to empirically find the chance of the approximation distracting the optimization process from the optimal point, the three algorithms were executed for ten thousand different position of different number of stations ($M \in [1, 25]$). Doing this experiment, only one erratic incident was found. With $M = 3$, $g_1 = 0.39 \times 10^{-13}$, $g_2 = 0.23 \times 10^{-13}$, and $g_3 = 0.05 \times 10^{-13}$, the approximation made an error in spotting the optimum point in CSC^a. This error induced a 3% deviation in the final result. Based on this experiment we can roughly state that there is a less than 0.1% chance for a less than 5% error when the approximation is used. However, note that the error has occured when no maximum

Table 3. The case in which approximation distracts the optimization process from finding the best solution

Solution		Station 1				Station 2				Station 3				Total	
k	x_k	x_1	$\hat{x}_1$	C_1	C_1^a	x_2	$\hat{x}_2$	C_2	C_2^a	x_3	$\hat{x}_3$	C_3	C_3^a	C	C^a
1	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.043	0.042
1	1.55	1.55	0.60	1.31	1.37	0.03	0.01	0.01	0.01	0.03	0.01	0.01	0.01	1.337	1.402
2	0.03	1.55	0.60	1.31	1.37	0.03	0.01	0.01	0.01	0.03	0.01	0.01	0.01	**1.337**	1.402
2	0.92	1.55	0.44	0.84	0.92	0.92	0.26	0.44	0.48	0.04	0.01	0.01	0.01	1.296	**1.413**
3	0.04	1.55	0.44	0.84	0.92	0.92	0.26	0.44	0.48	0.04	0.01	0.01	0.01	1.296	1.413
3	0.20	1.55	0.42	0.79	0.87	0.92	0.25	0.41	0.45	0.20	0.05	0.08	0.08	1.289	1.402

bound has been set for C_i. Here, we extensively analyze this case.

Table 3 analyzes this error. The bold number in the C row shows CSC's choice. Similarly, the bold number in C^a row show CSC^a's selection. The problem here is that the approximation has changed 1.296 into 1.413, carrying about 8% error. Then, mistakenly, CSC^a has rejected the value of 1.227 in delusion of having found a better value. Trying to find the root of this error, we see that C_1 and C_1^a are far apart, carrying more than 9% error. The reason for this error is the high $\hat{x}_1$ of about 0.44. Looking at Fig. 1b, we see that $\hat{x}_1$ is on the worst range in respect to the error of the approximation. One way to refrain from these errors is to have smaller $\hat{x}_i$'s, which means having more fair systems. In fact, in the case of NSC^a and N$^+$SC^a, we do have $\hat{x} \in [\varphi, \omega] = [0.03, 0.19]$, the shaded area in Fig. 1. This explains why the only case of erratic behavior observed here has happened in CSC^a and not NSC^a or N$^+$SC^a.

4. Conclusions

Based on the formulation of the reverse link aggregated capacity maximization problem available in the literature, a new approximation for computing the aggregate relative capacity was proposed. As the available algorithms find the aggregate capacity for a large set of candidate points, the approximation is beneficiary in reducing the overall computational cost. After giving mathematical guarantees that the application of the proposed approximation is acceptable within an error range, its actual implementation in the available algorithms is discussed. Also, it is shown that there is a decrease of one in orders of M in the computational costs of the available algorithms when the approximation is being used. Using examples and numerous safety checks, we observe that beyond a negligible probability, the approximation does not lead to false results. Analysis shows that there is a 0.1% possibility that there may be a less than 5% error in the results. Extensive investigation shows that this error happens in the case of the classical formulation of the problem, in which the system is capable of becoming very partial. This way, the approximation is shown to be vulnerable to monopoly of power. Hence, we conclude that, in more controlled environments, in which the share of powers of different stations are in a limited range, the approximation yields precise results while reducing the computational cost by a factor of more than $\frac{1}{5}M$, where M is the number of the stations.

Acknowledgments

The research of A.S. Alfa is partially supported by a grant from NSERC. The first author wishes to thank Ms. *Azadeh Yadollahi* for her encouragement and valuable discussions.

References

[1] Knopp R, Humblet P. Information capacity and power control in single-cell multiuser communications. In: 1995 IEEE International conference on communications, Seattle, 1995. p. 331–5.

[2] Hanly SV, Whiting P. Information-theoretic capacity of multi-receiver networks. Telecommun Syst 1993;1:1–42.

[3] Lee W. Estimate of channel capacity in Rayleigh fading environment. IEEE Trans Veh Technol 1990;39(3):187–9.

[4] Hanly SV, Tse D. Power control and capacity of spread-spectrum wireless networks. Automatica 1999;35(12): 1987–2012.

[5] Kandukuri S, Boyd S. Simultaneous rate and power control in multirate multimedia CDMA systems. In: IEEE sixth international symposium on spread spectrum and techniques applications, NJIT, NJ, 2000. p. 570–574.

[6] Oh S-J, Zhang D, Wasserman KM. Optimal resource allocation in multi-service CDMA networks. IEEE Trans Wireless Commun 2003;2:811–21.

[7] Jafar SA, Goldsmith A. Adaptive multirate CDMA for uplink throughput maximization. IEEE Trans Wireless Commun 2003;2:218–28.

[8] Oh S-J, Soong ACK. QoS-constrained information-theoretic sum capacity of reverse link CDMA systems. IEEE Trans Wireless Commun 2006;5(1):3–7.

[9] Abadpour A, Alfa AS, Soong AC. Closed form solution for QoS-constrained information-theoretic sum capacity of reverse link CDMA systems. In: Proceedings of the second ACM Q2SWinet 2006, Torremolinos, Malaga, Spain, 2006. p. 123–8.

[10] Abadpour A, Alfa AS, Soong AC. A more realistic approach to information-theoretic sum capacity of reverse link CDMA systems in a single cell. In: Proceedings of the IEEE international conference on communications (ICC 2007), Glasgow, Scotland, 2007.

[11] Abadpour A, Alfa AS, Soong AC. Capacity-share controlled information-theoretic sum capacity of reverse link single-cell CDMA systems. In: 2007 IEEE 65th vehicular technology conference (VTC2007 Spring), Dublin, Ireland, 2007.

[12] Goodman D, Mandayam N. Power control for wireless data. IEEE Pers Commun Mag 2000;7:48–54.

[13] Rappaport T, Milstein L. Effects of radio propagation path loss on DS-CDMA cellular frequency reuse efficiency for the reverse channel. IEEE Trans Veh Technol 1992;41(3): 231–42.

Arash Abadpour was born in 1979 in Tehran, Iran. He received a B.Sc. from Department of Control, Faculty of Electrical Engineering, Sharif University of Technology, Iran, in 2003 and an M.Sc. in Scientific Computation, Computer Sciences, from Department of Computer Sciences, Faculty of Mathematics Science, Sharif University of Technology, Iran, in 2005. He is currently a Ph.D. student in the Electrical and Computer Engineering Department, University of Manitoba, Canada, where he works on QoS-Constrained Information Theoretic Capacity of CDMA Systems. He is also a research assistant in Telecommunications Research Labs (TRLabs), Winnipeg. His research interests are in optimization, with emphasis on stochastic systems and models and pattern recognition.

Attahiru Sule Alfa NSERC Industrial Research Chair of Telecommunications and Professor, Department of Electrical and Computer Engineering at the University of Manitoba, Winnipeg, Manitoba, Canada. Dr. Alfa carries out research in the areas of queueing and network theories with applications mostly to telecommunication systems. He has also applied these theories to manufacturing and transportation and traffic systems in the past. His current research interests are in the area of wireless communication networks, mobility, Internet traffic, stochastic models, performance analysis, network restoration, and teletraffic forecasting models. He has contributed significantly in the area of matrix-analytic methods for stochastic models. He has published in several journals, and most recently in Stochastic Models, Queueing Systems Theory and Applications, Naval Research Logistics, IEEE Journal on Selected Areas in Communications, IEEE Transactions on Vehicular Technology, IEEE Transaction on Wireless, IEEE Transaction on Mobile Computing, IEEE Transaction on Communications, IEEE Transaction on Parallel and Distributed Systems, Performance Evaluation, Journal of Applied Probability, Advances in Applied Probability, Mathematics of Computations and Numerische Mathematik. He belongs to the following organizations: APEGM, IEEE, and INFORMS.

Anthony C.K. Soong received the B.Sc. degree in animal physiology and physics from the University of Calgary, and the B.Sc. degree in electrical engineering, the M.Sc. degree in biomedical physics and Ph.D. degree in electrical and computer engineering from the University of Alberta. He is currently a principal engineer for advance research and standards at Huawei Technologies Co. Ltd, in the US. He serves as the chair for 3GPP2 TSG-C NTAH (the next generation radio access network technology development group) and vice chair for 3GPP2 TSG-C WG3 (the physical layer development group for CDMA 2000). Prior to joining Huawei, he was with the systems group for Ericsson Inc. and Qualcomm Inc. His research interests are in statistical signal processing, robust statistics, wireless communications, spread spectrum techniques, multicarrier signaling, multiple antenna techniques and physiological signal processing.

Dr. Soong is a senior member of the IEEE. He has published numerous scientific papers and has over 30 patents granted or pending. He received the 2005 award of merit for his contribution to 3GPP2 and cdma2000 development. He has served on the technical program committee and has chaired at numerous major conferences in the area of communications engineering. He has acted as guest editor for the IEEE Communications Magazine and is a technical reviewer for the EURASIP Journal on Wireless Communications and Networking, IEEE Transactions on Communications, IEEE Transactions on Wireless Communications, IEEE Transactions on Vehicular Technology, IEEE Transactions on Signal Processing and IEEE Communication Letters.