UNIVERSITY OF MANITOBA

INFORMATION–THEORETIC SUM CAPACITY OF REVERSE LINK CDMA SYSTEMS IN A SINGLE CELL, A MORE REALISTIC APPROACH

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Summary

The information–theoretic approach to maximizing the aggregate capacity of the reverse link in a CDMA system looks for the best pattern of transmission power of the stations. In this framework, where the transmission from each station is noise to all others, extra constraints should be designed to lead to a practically applicable solution. The previous research has suggested a minimum guaranteed quality of service plus bounds on individual transmissions and the aggregate one. However, extensive analysis has revealed that these two constraints are not enough to produce a solution which can be realized in an actual system. Basically, lack of any constraint on neither the maximum capacity of each station nor the unfairness of the whole system has been found to be responsible for the partial solution in which all stations except for one are left to transmit at the lowest possible, while the selected station is served with a non-realistic bandwidth of couple of hundreds more. In this paper we devise a maximum capacity constraint and give an algorithm for solving the problem. Then, empirical evidence are analyzed to show that the system is actually becoming more even and practical when the new constraint is added.
Keywords: Quality of Service, Single Cell, Reverse Link CDMA, Optimization.

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1 Introduction

The first analysis of the system–wide information theoretic capacity of CDMA systems was given by *Knopp* and *Humblet* [1]. For a more comprehensive survey refer to [2]. Compared to these works which study the vector of all capacities of all the stations the analysis given in [3, 4, 5, 6] focus on the aggregate capacity. While different models are adopted in these works we select the Shannon–based formulation given in [4] because of its mathematical tractability. Also, see [2] for theoretical justification of this model. Here, we first discuss the problem and its formulation.

In [4] the authors developed a method for obtaining the maximum reverse link capacity of a CDMA system under some imposed quality of service constraints. The constraints of that problem included minimum signal to noise ratio, maximum and minimum bounds for transmission powers, and maximum bound on the aggregate transmission power. One of the main contributions of [4] is the minimum signal to noise ratio bound which was devised in order to solve the impracticality of the solution reported in [5, 6]. We will first briefly review the original problem. Then, we will show how another constraint is added to the problem to make the solution more practical.

Capacity of a single point–to–point communication link is well approximated by the Shannon theorem as $C = B\log_2\left(1+\frac{S}{N}\right)$ [2] (Also, see [3]). Here, B is the bandwidth and $\frac{S}{N}$ is the signal to noise ratio of the communication link. In the rest of this paper we omit B knowing that it is a constant multiplier. Hence, we are looking at relative capacities. Assume that there are M mobile stations whose reverse link gains are $g_1, \cdots, g_M$, satisfying $g_1 > \cdots > g_M$. Define the i–th mobile station's transmit power as p_i, for which we have $0 \leq p_i \leq p_{max}$. With a background noise of I, the signal to noise ratio for the i–th station becomes,

$$\gamma_i = \frac{p_i g_i}{I + \sum_{j=1, j\neq i}^{M} p_j g_j}, \tag{1}$$

Hence, using Shannon formula the relative capacity of the i–th station becomes,

$$C_i = \log_2(1+\gamma_i) = \log_2 \frac{I + \sum_{j=1}^{M} p_j g_j}{I + \sum_{j=1, j\neq i}^{M} p_j g_j}. \quad (2)$$

Using (2) for computing the aggregate capacity of the system, we have,

$$C(\vec{\mathbf{p}}) = \log_2 \frac{\left(I + \sum_{j=1}^{M} p_j g_j\right)^M}{\prod_{i=1}^{M}\left(I + \sum_{j=1, j\neq i}^{M} p_j g_j\right)}. \quad (3)$$

Here, $\vec{\mathbf{p}} = (p_1, \cdots, p_M)$. Adding $\forall i, \gamma_i \geq \gamma$ and $\sum_{i=1}^{M} p_i g_i \leq P_{max}$ the classical single–cell problem is shaped up as finding $\vec{\mathbf{p}}$ which gives,

$$\max \frac{\left(I + \sum_{j=1}^{M} p_j g_j\right)^M}{\prod_{i=1}^{M}\left(I + \sum_{j=1, j\neq i}^{M} p_j g_j\right)}, \quad (4)$$

given,

$$\forall i, 0 \leq p_i \leq p_{max}, \quad (5)$$

$$\forall i, \frac{p_i g_i}{I + \sum_{j=1, j\neq i}^{M} p_j g_j} \geq \gamma, \quad (6)$$

$$\sum_{i=1}^{M} p_i g_i \leq P_{max}. \quad (7)$$

While the search space for the classical problem is $[0, p_{max}]^M$, of dimension M, *Oh* and *Soong* proved that the search can be limited to a multiply of M number of one–dimensional intervals. For that they utilized a numerical optimization method [4]. Then, *Abadpour*, *Alfa*, and *Soong* showed that the search can actually be limited to less than $2M$ points. In this way, the computational cost of the classical problem was found to be $O(M^2)$ [7]. The analysis given in [7] also showed that the solution to the classical single–cell problem is very prone to becoming drastically partial. In an extensive empirical review it was observed that the solution always included every station being treated with the lowest possible bandwidth except for only one which was served at a bandwidth couple of hundreds as much as the others. The argument was then that this situation may not be practically useful when in the real life there may be no station capable of handling such a huge bandwidth. So, the conclusion of [7] was that the classical problem have to be reformulated to include a maximum bound on the capacities offered to single stations. Also, it was suggested that the unfairness should be more seriously analyzed.

In this paper we include a maximum bound on the capacity of each station and use the same mathematical tools introduced in [7] to solve the new problem. Also, we will investigate the expected range of unfairness prior to solution. We will show that while a maximum bound increases the practicality of the problem to high extents it only increases the computational cost to $O(M^3)$.

The rest of this paper is organized as follows. First, the proposed method is given in Section 2. Then, Section 3 holds the experimental results and finally Section 4 concludes the paper.

2 Proposed Method

This section is organized as follows. First, Section 2.1 briefly reviews the mathematical approach introduced in [7] to solve the classical single–cell problem. Then, in Section 2.2 the new maximum capacity constraint is introduced and it is shown how it integrates into the whole problem. Then, in Section 2.3 we show how the new problem is similar to the classical one. As such, we justify that properties similar to what was proved before for the classical problem, are still applicable to the new problem. Section 2.4 uses these results to propose an algorithm to solve the new problem. Then, Section 2.5 discusses the computational cost of the proposed algorithm. Finally, Section 2.6 shows how the new constraint puts a limit on the unfairness of the system and distributes the resources in the system.

2.1 Mathematical Method

This section briefly reviews the mathematical approach and the tools developed in [7] for solving the classical single–cell problem. We will use these tools for solving the new problem.

Using the linear transformation,

$$x_i = \frac{p_i g_i}{I}, \tag{8}$$

the classical problem reduces to minimizing,

$$\Phi(\vec{\mathbf{x}}) = \frac{\prod_{i=1}^{M}\left(1+\sum_{j=1}^{M} x_j - x_i\right)}{\left(1+\sum_{j=1}^{M} x_j\right)^M}. \tag{9}$$

given that,

$$\forall i, 0 \leq x_i \leq l_i = \frac{p_{max}}{I} g_i, \tag{10}$$

$$\forall i, \frac{x_i}{1+\sum_{j=1}^{M} x_j} \geq \varphi = \frac{\gamma}{\gamma+1}, \tag{11}$$

$$\sum_{i=1}^{M} x_i \leq X_{max} = \frac{P_{max}}{I}. \tag{12}$$

Also, it was shown that investigating the behavior of the single–cell problem in specific hyperplanes is beneficial. Hence, the single–cell problem was investigated in the hyperplane,

$$\sum_{i=1}^{M} x_i = T, \tag{13}$$

for different values of T. In these hyperplanes the constraints have simpler formulations. This way, the bound for the aggregate transmission power simply changes into a limitation for T as $T \leq X_{max}$. Also, (11) changes into $\forall i, x_i \geq \varphi(1+T)$, which, in combination with (10) gives,

$$\forall i, \varphi(1+T) \leq x_i \leq l_i. \tag{14}$$

It was proved that if we can limit the search space to $\prod_{i=1}^{M}[b, B_i]$ where b and B_is are positive values and B_is are sorted in a descending fashion then the optimum solution has a very interesting structure. Actually, it was proved that the minimum of $\Phi(\vec{\mathbf{x}})$ in this search space happens when $\vec{\mathbf{x}}$ has a structure like,

$$\vec{\mathbf{x}} = (B_1, \cdots, B_{k-1}, x_k, b, \cdots, b)\,. \tag{15}$$

It was proved that for the function $f(x)$ defined as,

$$f(x) = \frac{\prod_{i=1}^{k}(x+\beta_i)}{(x+\beta)^k}, \tag{16}$$

where $0 < \beta_1 < \beta_2 < \cdots < \beta_{k-1} < \beta < \beta_k$ and $\sum_{i=1}^{k}\beta_i > k\beta$, the minimum of $f(x)$ for $x \in [a,b] \subset R^+ \cup \{0\}$ happens either in a or b.

2.2 Maximum Capacity Bound

First let's discuss the minimum quality of service bound given in (6). Using (2), the constraint given in (6) gives $C_i \geq \log_2(1+\gamma)$. Hence, having $\gamma + 1 = (1-\varphi)^{-1}$, (6) becomes $C_i \geq -\log_2(1-\varphi)$. This equation shows that the value of φ, and equivalently γ, determine a minimum value for the capacity of each station. Using Taylor series we know that for the small values of x, $\log_2(1-x) \simeq -\frac{1}{\ln(2)}x$. Hence, we have, $C_i \geq \frac{1}{\ln(2)}\varphi$. This means that φ is directly a minimum bound for the capacity of each station.

According to the discussions given at the beginning of this paper, we propose to add a new constraint to those given in (5), (6), and (7). Namely, we propose this new constraint,

$$\forall i, C_i \leq \eta. \tag{17}$$

We will show that this constraint will actually control the unfairness of the solution. Let's work on (17). According to (2), the new constraint says,

$$\forall i, \log_2 \frac{1+\sum_{j=1}^{M} x_j}{1+\sum_{j=1, j\neq i}^{M} x_j} \leq \eta. \tag{18}$$

Note that we are using the linear transformation of the search–space discussed in Section 2.1. Hence,

$$\forall i, \frac{1+T}{1+T-x_i} \leq 2^{\eta}. \tag{19}$$

Here, we have used (13). Continuing with (19) we have $\forall i, x_i \leq (1-2^{-\eta})(1+T)$. Now, defining,

$$\omega = 1 - 2^{-\eta}, \tag{20}$$

We have, $\forall i, x_i \leq \omega(1+T)$, which is very much like the left side of (14) except for the fact that here we are putting an upper bound on x_i, compared to the lower bound given in (14). Note that $\omega > \varphi$ is a necessary condition for the existence of any solution.

2.3 Spotting the Solution

In this framework, the problem is minimizing,

$$\Phi(\vec{\mathbf{x}}) = \frac{\prod_{i=1}^{M}\left(1+\sum_{j=1}^{M} x_j - x_i\right)}{\left(1+\sum_{j=1}^{M} x_j\right)^{M}}. \tag{21}$$

given that,

$$\forall i, 0 \leq x_i \leq l_i, \tag{22}$$

$$\forall i, \varphi \leq \frac{x_i}{1+\sum_{j=1}^{M} x_j} \leq \omega, \tag{23}$$

$$\sum_{i=1}^{M} x_i \leq X_{max}. \tag{24}$$

To solve this problem we use the same method of performing the analysis in specific hyperplanes discussed briefly in Section 2.1 (refer to [7] for details). As the structural arrangement discussed in Section 2.1 is still valid, so we can use the Essential Theorem here, too. Note that in this new situation, for some values of i, the maximum bound of $x_j, j \geq i$ is $\omega(1+T)$ and not l_j. It is clear that the same pyramid–like structure of the search space, the descending fashion of upper–bounds, is still happening. Hence, in accordance with (15), we infer that the optimal solution to the single–cell problem when the new constraint is included, is like,

$$\vec{\mathbf{x}} = \Big(\omega(1+T), \cdots, \omega(1+T), l_{j+1}, \cdots, l_{k-1}.x_k, \varphi(1+T), \cdots, \varphi(1+T)\Big). \tag{25}$$

We remember the structure of the solution to the classical single–cell problem which was as,

$$\vec{\mathbf{x}} = (l_1, \cdots, l_{k-1}, x_k, \varphi(1+T), \cdots, \varphi(1+T)). \tag{26}$$

Basically (25) is very similar to (26) except for the fact that no x_i is allowed to exceed a certain limit. Comparing (25) with (26) reveals that in the new problem there are two values of j and k which have to be found while in the last problem we were only faced with k [7]. Hence, it would be understandable if solving this problem would be of a higher order. We will analyze the computational cost of this algorithm in Section 2.5. From (25) we know that,

$$T = \sum_{i=1}^{M} x_i = j\omega(1+T) + \sum_{i=j+1}^{k-1} l_i + (M-k)\varphi(1+T). \tag{27}$$

Hence,

$$1 + T = \frac{x_k + \sum_{i=j+1}^{k-1} l_i + 1}{1 - [j\omega + (M-k)\varphi]}. \tag{28}$$

Defining,

$$L = \sum_{i=j+1}^{k-1} l_i, \tag{29}$$

and,

$$\psi = 1 - [j\omega + (M-k)\varphi], \tag{30}$$

we have,

$$1 + T = \frac{1}{\psi}(x_k + L + 1). \tag{31}$$

Note that both ψ and L depend on j and k but not on x_k.

Trying to fulfill the constraints for different values of i, we reach to these necessary inequalities.

$$1 + T \leq X_{max} + 1, \tag{32}$$

$$\omega \geq \varphi, \tag{33}$$

$$j \neq 0 \rightarrow l_j \geq \omega(1 + T), \tag{34}$$

$$l_M \geq \varphi(1 + T), \tag{35}$$

$$k > j + 1 \rightarrow l_{j+1} \leq \omega(1 + T), \tag{36}$$

$$x_k \leq l_k, \tag{37}$$

$$x_k \leq \omega(1 + T), \tag{38}$$

and,

$$x_k \geq \varphi(1 + T). \tag{39}$$

Working on (32) we have,

$$\frac{1}{\psi}(x_k + L + 1) \leq X_{max} + 1 \rightarrow x_k \leq \psi(X_{max} + 1) - (L + 1). \tag{40}$$

Note that here we have used (31). Also, (34) yields,

$$l_j \geq \omega\frac{1}{\psi}(x_k + L + 1) \rightarrow x_k \leq \frac{\omega}{\psi}l_j - (L + 1). \tag{41}$$

Note that this condition is only necessary if j is not zero. Using the notation $[P]$ where P is a boolean argument we can write it as,

$$x_k \leq \left(\frac{\omega}{\psi}l_j - (L + 1)\right)\frac{1}{1 - [j = 0]}. \tag{42}$$

Here, $[P]$ is one if the condition P holds and zero otherwise. By this notation we mean that the condition depicted in (42) changes to the obvious $x_k \leq \infty$ for $j = 0$.

From (35) we have,

$$l_M \geq \varphi\frac{1}{\psi}(x_k + L + 1) \rightarrow x_k \leq \frac{\psi}{\varphi}l_M - (L + 1). \tag{43}$$

Also, (36) results in,

$$l_{j+1} \leq \omega\frac{1}{\psi}(x_k + L + 1) \rightarrow x_k \geq \frac{\psi}{\omega}l_{j+1} - (L+1), \tag{44}$$

if $k > j + 1$. Equivalently,

$$x_k \geq \left(\frac{\psi}{\omega}l_{j+1} - (L+1)\right)(1 - [k \leq j+1]). \tag{45}$$

Note that in the case of $k \leq j + 1$ inequality (45) changes into the trivial $x_k \geq 0$. Equation (38) dictates,

$$x_k \leq \omega(1+T)\frac{1}{\psi}(x_k + L + 1) \rightarrow \left(1 - \frac{\omega}{\psi}\right)x_k \leq \frac{\omega}{\psi}(L+1). \tag{46}$$

If $\psi < \omega$ then this inequality is trivial. Otherwise,

$$x_k \leq \frac{\omega}{\psi - \omega}(L+1). \tag{47}$$

So we write,

$$x_k \leq \frac{\omega}{\psi - \omega}(L+1)\frac{1}{1 - [\psi < \omega]}. \tag{48}$$

Finally, (39) gives,

$$x_k \geq \varphi\frac{1}{\psi}(x_k + L + 1) \rightarrow x_k \geq \frac{\varphi}{\psi - \varphi}(L+1), \tag{49}$$

which also needs,

$$\psi > \varphi. \tag{50}$$

From (37), (40), (42), (43), (45), (48), (49) and (50) we reach to these necessary and sufficient conditions.

$$\psi > \varphi, \tag{51}$$

$$x_k \leq \min\left\{l_k, \frac{\omega}{\psi - \omega}(L+1)\frac{1}{1 - [\psi < \omega]}, \psi \min\left\{\frac{1}{\omega}l_j\frac{1}{1 - [j = 0]}, \frac{1}{\varphi}l_M, X_{max} + 1\right\} - (L+1)\right\}, \tag{52}$$

$$x_k \geq \max\left\{\left(\frac{\psi}{\omega}l_{j+1}-(L+1)\right)(1-[k\leq j+1]),\frac{\varphi}{\psi-\varphi}(L+1)\right\}. \tag{53}$$

It is now interesting to look at the case of large η. Assuming $\eta=\infty$ leads to $\omega=1$. This case means the maximum constraint is actually removed. Using (30) we have,

$$\psi-\omega=-(M-k)\varphi<0, \tag{54}$$

and also $j=0$. Hence, this case results in (52) being converted to,

$$x_k \leq \min\left\{l_k,(1-(M-k)\varphi)\min\left\{\frac{1}{\varphi}l_M,X_{max}+1\right\}-(L+1)\right\}, \tag{55}$$

which is identical to the maximum limit in the classical case [7]. Also, (53) changes to,

$$x_k \geq \max\left\{\Big((1-(M-k)\varphi)\,l_1-(L+1)\Big)(1-[k\leq j+1]),\frac{\varphi(L+1)}{1-(M-k+1)\varphi}\right\}, \tag{56}$$

which seems to differ from the classical case [7]. However, a closer look reveals that if $k\geq 2$ then using (29),

$$L=\sum_{i=1}^{k-1}l_i \geq l_1 \geq (1-(M-k)\varphi)\,l_1. \tag{57}$$

Hence, the first term in (56) is actually negative. In contrary, if $k<2$ then $k<j+1$ which eliminates the first condition. Hence, (56) converts to,

$$x_k \geq \frac{\varphi(L+1)}{1-(M-k+1)\varphi}, \tag{58}$$

which is exactly the one used in the classical case [7]. So, as expected, as η tends to infinity the new problem converts to the classical one.

Now, let's work on the objective function. Substituting (25) into (21) we have,

$$\Phi(\vec{\mathbf{x}})=\left(\frac{1+T-\omega(1+T)}{1+T}\right)^{j}\prod_{i=j+1}^{k-1}\left(\frac{1+T-l_i}{1+T}\right)\left(\frac{1+T-x_k}{1+T}\right)\left(\frac{1+T-\varphi(1+T)}{1+T}\right)^{M-k}. \tag{59}$$

Now, using (31) we have,

$$\Phi(\vec{\mathbf{x}}) = \frac{c\psi}{1-\psi} \prod_{i=j+1}^{k-1} \left(\frac{\frac{1}{\psi}(x_k + L + 1) - l_i}{\frac{1}{\psi}(x_k + L + 1)} \right) \left(\frac{\frac{1}{\psi}(x_k + L + 1) - x_k}{\frac{1}{\psi}(x_k + L + 1)} \right), \quad (60)$$

Here, c is a constant defined as, $c = \frac{1-\psi}{\psi}(1-\omega)^j(1-\varphi)^{M-k}$. Continuing with (60) we have,

$$\Phi(\vec{\mathbf{x}}) = c\frac{\prod_{i=1}^{n}(x_k + \beta_i)}{(x_k + \beta)^n}. \quad (61)$$

Here, we have $n = k - j$ and,

$$\begin{cases} \beta_i = \begin{cases} L + 1 - \psi l_{j+i} & i = 1, \cdots n-1 \\ \frac{L+1}{1-\psi} & i = n \end{cases} \\ \beta = L + 1 \end{cases}. \quad (62)$$

Notice that here we have $0 < \beta_1 < \beta_2 < \cdots < \beta_{n-1} < \beta < \beta_n$. Also,

$$\sum_{i=1}^{n} \beta_i - n\beta = (n-1)(L+1) - \psi L + \frac{L+1}{1-\psi} - n(L+1) = \frac{\psi(\psi L + 1)}{1-\psi} > 0. \quad (63)$$

Hence, according to the theorem given in Section 2.1 we infer that the minimum value of Φ happens when x_k accepts one of the boundary values given in (52) and (53).

2.4 Proposed Algorithm

In this section, we integrate the results of the last sections and propose a new algorithm for solving the single–cell problem.

- **Aim** Finding the distribution of transmit powers of M mobile stations which gives the maximum aggregate system capacity.
- **Inputs**
  1. Number of stations M.
  2. Positive values of $g_1 > \cdots > g_M$.
  3. Background noise I.
  4. Minimum quality of service γ.
  5. Maximum relative capacity η.
  6. Maximum power of one station p_{max}.
  7. Maximum aggregate power P_{max}.

- **Outputs**
  1. Transmit power of stations $p_1, \cdots, p_M$.
  2. Relative capacities of stations $C_1, \cdots, C_M$.
  3. Aggregate relative capacity C.
  4. Measures of unfairness f and $\tilde{f}$.

- **Algorithm**
  1. Compute X_{max} as (12),$X_{max} = \frac{P_{max}}{I}$.
  2. Compute φ as (11), $\varphi = \frac{\gamma}{\gamma+1}$.
  3. Compute ω as (20), $\omega = 1 - 2^{-\eta}$ and report "*Error*" if $\omega < \varphi$.
  4. For all $1 \leq i \leq M$ compute l_i as (10), $l_i = \frac{p_{max}}{I} g_i$.
  5. For all j from 0 to M and for all k from $j+1$ to M do the followings,
     (a) Compute ψ as (30), $\psi = 1 - [j\omega + (M-k)\varphi]$.
     (b) If not $\psi > \varphi$ then go to Line 5 and start over for new values of j and k, else continue to Line 5c.
     (c) Compute L as (29), $L = \sum_{i=j+1}^{k-1} l_i$.
     (d) Compute max_x as (52), $max_x = \min\left\{l_k, \frac{\omega}{\psi-\omega}(L+1)\frac{1}{1-[\psi<\omega]}, \psi \min\left\{\frac{1}{\omega} l_j \frac{1}{1-[j=0]}, \frac{1}{\varphi} l\right.\right.$
     (e) compute min_x as (53), $min_x = \max\left\{\left(\frac{\psi}{\omega} l_{j+1} - (L+1)\right)(1-[k \leq j+1]), \frac{\varphi}{\psi-\varphi}(L\right.$
     (f) If not $max_x \geq min_x \geq 0$ then go to Line 5 and start over for new values of j and k, else continue to Line 5g.
     (g) Do the following lines for two values of $x_k = min_x$ and $x_k = max_x$, separately. Store both ϕ and the values of x_i for each trial.
        - Compute T as (31), $1 + T = \frac{1}{\psi}(x_k + L + 1)$.
        - Set $x_i = \omega(1+T)$ for $i = 1, \cdots, j$.
        - Set $x_i = l_i$ for $i = j+1, \cdots, k-1$.
        - Set $x_i = \varphi(1+T)$ for $i = k+1, \cdots, M$.
        - Compute ϕ as (21), $\phi = \prod_{i=1}^{M}\left(1 + \sum_{j=1}^{M} x_j - x_i\right)\left(1 + \sum_{j=1}^{M} x_j\right)^{-M}$.
  6. Find the smallest ϕ produced at the above and retrieve the corresponding values of x_i.
  7. For the specified values of x_i find C_i using (2), $C_i = \log_2\left(1 + \sum_{j=1}^{M} x_j\right)\left(1 + \sum_{j=1}^{M} x_j -\right.$
  8. Compute f as (64), $f = \max\{C_i\} - \min\{C_i\}$.
  9. Compute $\tilde{f}$ as (65), $\tilde{f} = \frac{\max\{C_i\}}{\min\{C_i\}}$.
  10. Compute p_i using (8), $p_i = \frac{I x_i}{g_i}$, and return them.
  11. Return C computed as, $C = -\log_2 \phi$.

2.5 Computational Cost

Analysis of the computational cost of the algorithm depicted in Section 2.4 shows that it demands $\tau = \frac{16}{3}\varepsilon M^3 + \frac{53\varepsilon+2}{3}M^2 - 23\varepsilon M + 6$ flops. Here, ε is the portion of trials for which ϕ is actually computed. The worst case, setting $\varepsilon = 1$, gives, $\tau = \frac{16}{3}M^3 + \frac{55}{3}M^2 - 23M + 6$ flops. However, for a typical system[1] ε is 0.2. On a 1Gfps (e.g. TMS320C6713B from *Texas Instruments*), it takes less than 6ms to do the calculations of the proposed algorithm when M equals 100 (5.5ms and 0.6ms for the worst case and the bast one). Note that here the computational cost is of order $O(M^3)$ compared to the $O(M^2)$ algorithm given for the classical case [7]. Actually this was expected because the main "*for*" loop here contains two variables each one ranging somewhere between 0 and M. In Section 3 we show that the acceptable results of this algorithm compensates for its higher computational cost. As a rule of thumb note that the computational cost of the new algorithm is almost $\frac{2}{3}M$ times the classical one.

2.6 Fairness Analysis

Here, we investigate two unfairness measures of subtractive unfairness,

$$f = \max\{C_i\} - \min\{C_i\}, \tag{64}$$

and ratio unfairness,

$$\tilde{f} = \frac{\max\{C_i\}}{\min\{C_i\}}. \tag{65}$$

Using (17) and (20) we know that $\forall i, C_i \leq \eta = -\log_2(1-\omega)$ which gives, $\max\{C_i\} \leq -\log_2(1-\omega)$. Also, we have, $\forall i, C_i \geq -\log_2(1-\varphi)$, resulting in, $\min\{C_i\} \geq -\log_2(1-\varphi)$. Hence,

$$f \leq -\log_2(1-\omega) + \log_2(1-\varphi) = \log_2\left(\frac{1-\varphi}{1-\omega}\right). \tag{66}$$

[1]Parameters are given as $\gamma = -30dB$, $I = -113dBm$, $P_{max} = -106dBm$, $p_{max} = 23dBm$, $M = 10$, $\eta = 0.2$ [8].

This equation has an interesting meaning. Namely, after we select φ, which indicates the minimum guaranteed quality of service, we can find the appropriate ω, and hence η, which result in a desired unfairness. After all, we have shown that the system's unfairness is controlable when the new constraint is added.

For the second unfairness measure define in (65) we have,

$$\tilde{f} = \frac{\max\{C_i\}}{\min\{C_i\}} \leq \frac{-\log_2(1-\omega)}{-\log_2(1-\varphi)}. \tag{67}$$

Note that both terms $-\log_2(1-\omega)$ and $-\log_2(1-\varphi)$ are positive. Again, we see how the new constraint puts a limit on the ratio unfairness.

Using (20) and (11) we give handy approximations for f and $\tilde{f}$. Here, we use the notations $f^\star$ and $\tilde{f}^\star$ for the maximum expected subtractive and ratio unfairness measures, respectively, given γ and η. Having these two values we have guaranteed maximum values for f and $\tilde{f}$. From (66) we have,

$$f^\star = \log_2\left(\frac{1-\varphi}{1-\omega}\right) = \log_2\left(\frac{2^\eta}{1+\gamma}\right) = \eta - \log_2(1+\gamma) \simeq \eta - \frac{1}{\ln 2}\gamma, \tag{68}$$

Here, we have used the fact that if γ is very small then $\ln(1+\gamma) \simeq \gamma$. Equation (68) shows that the maximum possible subtractive unfairness is a linear function of η and γ. Although, as expected, γ has a negative contribution to $f^\star$ (with a coefficient of about 1.44), according to the fact that γ is so small, we know that η is the main factor which determines $f^\star$. We also give an approximation for (67) as,

$$\tilde{f}^\star = \frac{\log_2(1-\omega)}{\log_2(1-\varphi)} = \frac{\eta}{\log_2(1+\gamma)} \simeq \frac{\eta}{\gamma}\ln 2. \tag{69}$$

The two equations (68) and (69) relate $f^\star$ and $\tilde{f}^\star$ to γ and η. However, we may like to determine $f^\star$ and $\tilde{f}^\star$ and then want to have a way to determine respective values of γ and η. Solving (68) and (69) for γ and η we have,

$$\gamma = 2^{\frac{f^\star}{\tilde{f}^\star - 1}} - 1 \simeq \frac{f^\star}{\tilde{f}^\star - 1}\ln 2, \tag{70}$$

and,

$$\eta = \frac{f^\star \tilde{f}^\star}{\tilde{f}^\star - 1}. \tag{71}$$

Note that $f^\star \geq 0$ and $\tilde{f}^\star \geq 1$, by definition.

As a numerical example having $\gamma = -50dB$ and $\eta = 0.3$ we will have $f^\star = 0.30$ and $\tilde{f}^\star = 65.76$. Compared to Table 1, where $f = 0.25$ and $\tilde{f} = 6.02$, we will infer that $f^\star$ is actually a good approximation. Also, while $\tilde{f}^\star$ is still a conservative value the actual value gets far better. Coming from the other side, selecting $f^\star = 5$ and $\tilde{f}^\star = 0.2$ yields $\gamma \simeq -30dB$ and $\eta = 0.25$. We can also do it like this, what is the value of η to make $\tilde{f}^\star = 3$ with $\gamma = -50dB$. The answer is $\eta = 0.014$. Then, we will also have $f^\star = 0.0091$.

3 Experimental Results

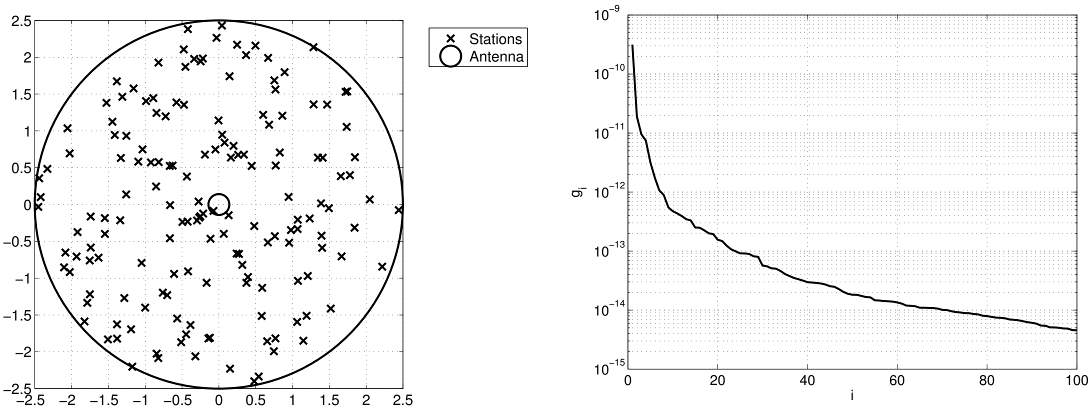
The proposed algorithm for solving a single–cell problem is developed in MATLAB 7.0.4 on a PIV 3.00GHZ with 1GB of RAM. In this framework, it takes less than $2ms$ to give the solution for a cell containing 100 stations.

The parameters of the problem are the sequence $g_1, \cdots, g_M$ plus I, γ, p_{max}, P_{max}, and η. To compare the effects of different parameters, a sample sequence g_i is produced using a set of random numbers that comply with the conditions. To be able to compare the results for different values of M having a controlled variation, we down–sample this sequence to produce new sequences with different lengths (to analyze the effect of number of stations). This way we can compare the effect of selecting different values of different parameters.

The base parameters used in this study are [8, 9],

$$\gamma = -30dB, I = -113dBm, P_{max} = -106dBm, p_{max} = 23dBm, M = 10, \eta = 0.3. \tag{72}$$

Whenever a parameter is different from the list given in the above it is mentioned.



(a) (b)

Figure 1: (a) Location of different stations in one cell. (b) Sequence of reverse gains.

The conversion from dB to a real value is done according to $xdB \equiv 10^{\frac{1}{20}x}$. Also, $xdBm \equiv 10^{\frac{1}{10}x} mw$.

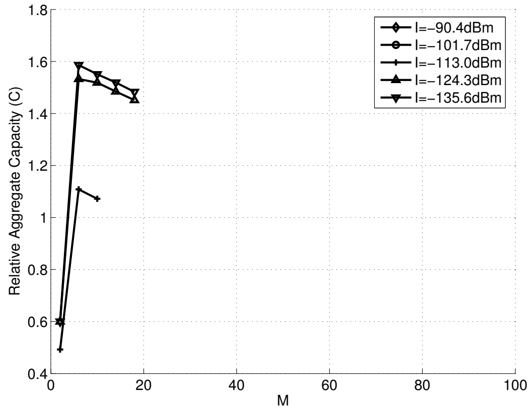
Here, we work on a circular cell of radius $R = 2.5Km$. For the station i at the distance d_i from the base station we only assume the path gain which is calculated as $g_i = Cd_i^n$. Here, C and n are constants equal to $7.75 \times 10^{-}3$ and -3.66 when d_i is in meters. Equivalently, with d_i in kilometers we will have $C = 1.2283 \times 10^{-13}$. To produce M values of g_i we uniformly put $3M$ points in the $[-R, R] \times [-R, R]$ and from those in the circle with radius R centered at the origin we select M points. Figures 1–a and 1–b show the location of different stations and the sequence of reverse gains, respectively. The surrounding circle in Figure 1–b shows the border of the cell.

Table 1: Comparison of the classical problem with the new one. [P: Pattern. Here, x and X mean the station is transmitting with the minimum and maximum capacities, respectively. Also, b and l mean x_i is in between or equals l_i, respectively. g_i: Reverse Link. p_i: Power (mw). C_i: Relative Capacity. $\tilde{C}_i$: Capacity Share. C: Aggregate Capacity. f: Subtractive Unfairness. $\tilde{f}$: Ratio Unfairness.]

Station #		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
g_i ($\times 10^{-12}$)		0.52	0.018	0.016	0.0091	0.0082	0.0081	0.0075	0.0059	0.0059	0.0045
	P	b	x	x	x	x	x	x	x	x	x
	p_i	46.5266	5.2114	5.8548	10.4390	11.6260	11.7556	12.6169	16.0532	16.1979	20.9157
Classical	C_i	2.3606	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046
Problem [7]	$\tilde{C}_i$	98.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%
	C	2.402									
	f	2.356									
	$\tilde{f}$	518.2									
	P	X	l	l	l	l	l	l	l	l	b
	p_i	14.662	16.092	16.189	16.584	28.796	36.313	50.000	13.744	14.535	18.984
New	C_i	0.3000	0.2111	0.1863	0.1015	0.0908	0.0898	0.0835	0.0652	0.0646	0.0498
Problem	$\tilde{C}_i$	24.1%	17.0%	15.0%	8.2%	7.3%	7.2%	6.7%	5.2%	5.2%	4.0%
	C	1.243									
	f	0.2502									
	$\tilde{f}$	6.027									

To give a comparison between the classical single–cell problem and its new version, we show the outcome of adding the new constraint on a sample problem. Table 1 shows the values of g_i for a sample example where M is equal to 10. Table 1 also shows the solutions to the classical and the new problems with the set of parameters given in (72). For each problem, first the pattern of the solution is given. According to the pattern of the solution to the classical and the new single–cell problems given in (26) and (25), respectively, we are interested to see where the breaking points are eventually placed. The location of these points informs us of the number of stations benefiting from higher capacities. As expected from the results and discussions given in [7], the classical solution serves the first station with the maximum possible capacity while the others are left to the minimum guaranteed ammount ($\log_2(1+\gamma)$). When we put the extra $\forall i, C_i \leq \eta$ constraint, we are actually limiting the first station's capacity. As seen in the results of the new problem, this results in a spread of the capacity between more users. In the new solution shown in Table 1, nine users are served at the maximum possible capacity (either determined by η or l_i) while the other one is in between. To show this more clearly, we define the capacity share of the i–th station as, $\tilde{C}_i = \frac{C_i}{C}$. Table 1 also shows the capacity share of different stations in both problems. See how the classical problem depends dominantly on the first station, while the new problem's dependency on different stations is more even. The high dependence of the classical problem on one station obviously is not an acceptable practice if the system is to be deployed in the real world. As expected both from the analysis and from the set of capacity values, the new problem is more fair than the classical one. In fact the ratio unfairness measure is now about eighty times lower in the new problem compared to the classical one. Note that here we see the aggregate capacity becoming smaller in the new problem set. This was expected, because by putting the maximum capacity constraint, we have actually limited the objective function from going up–hill. By this, we have gained a more fair system. Here, we see the problem of the classical problem in details. Basically, the solution given by the classical problem will not be implementable if the first station denies the high capacity which we have decided for it. In contrary, the new problem serves nine stations with the highest possible capacity and then serves one with a midpoint capacity. With this fair pattern of solution, the new problem is capable of supporting the aggregate capacity of above half the classical one.

Using the sequence g_i shown in Figure 1–b we analyze the effects of different parameters on the solution. The analysis is conducted in this way, first we show the values of C, the aggregate relative capacity. Then, we analyze the unfairness measures f and $\tilde{f}$. The first experiment analyzes how selecting between different values of I and γ affects the system while the number of stations changes from one to a hundred. It was shown in [7] that as γ decreases the aggregate capacity increases. The interpretation for that effect was that decreasing γ means that the system is less engaged with the majority and can allocate more resources to the elite station. It was argued that this is not practically an acceptable attitude. Figures 2, 3, 4, and 5 show that the new problem does not show an event like that. Here, γ has actually negligible effect on the aggregate capacity, partly because not many stations are treated at that low rate. Also, we see that with the new problem the aggregate capacity does not drop vastly when M increases. For example, looking at Figures 5–(a) we see that the new problem's aggregate capacity drops less than 0.1 when M goes from 1 to 100 (6.4% of the amplitude). At the same time the classical problem results in the relative aggregate capacity's drop by 6 (75% of the amplitude) [7]. However, like the case of the classical problem we see that increasing I decreases the aggregate capacity. It should be mentioned that the aggregate capacity of the new problem is almost one third of that of the classical problem and the difference increases vastly



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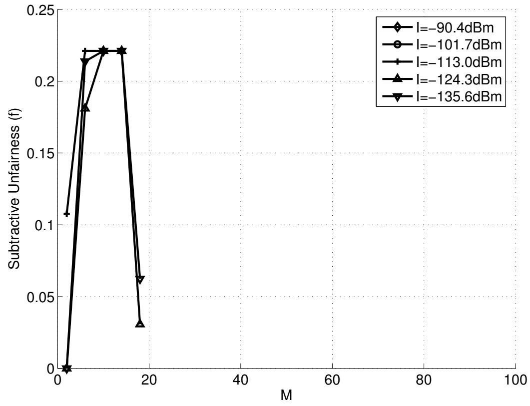

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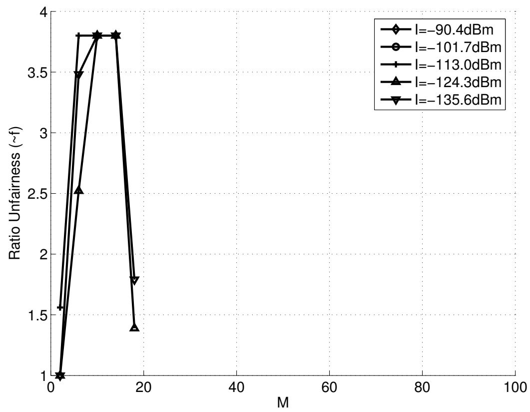
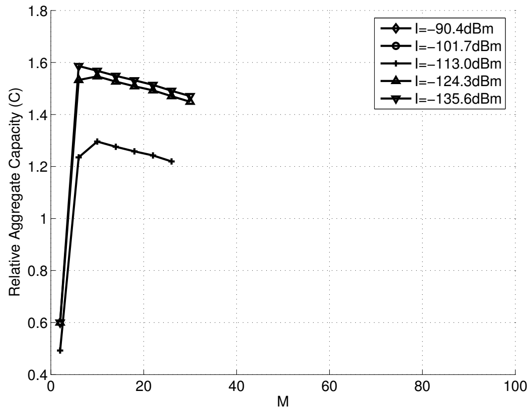


(c)

Figure 2: Results for different number of stations with varying I and $\gamma = -25dB$ in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



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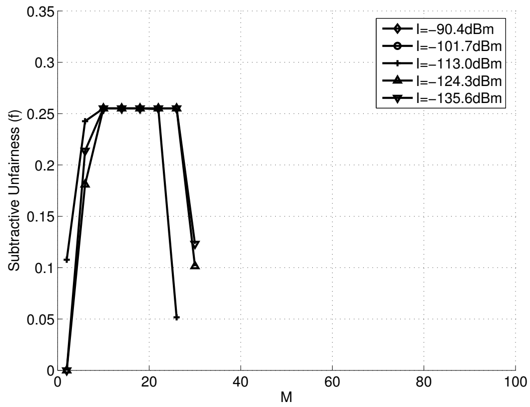

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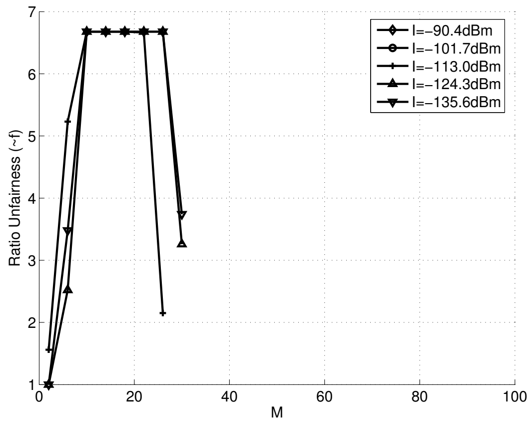
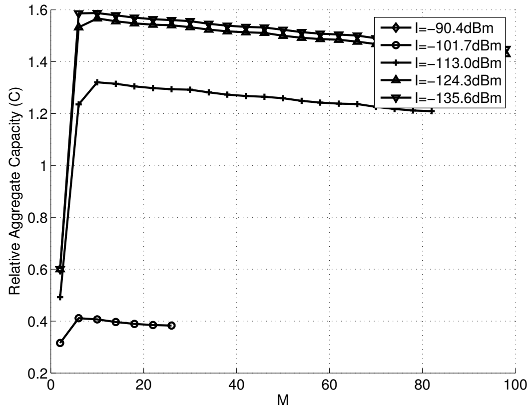


(c)

Figure 3: Results for different number of stations with varying I and $\gamma = -30dB$ in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



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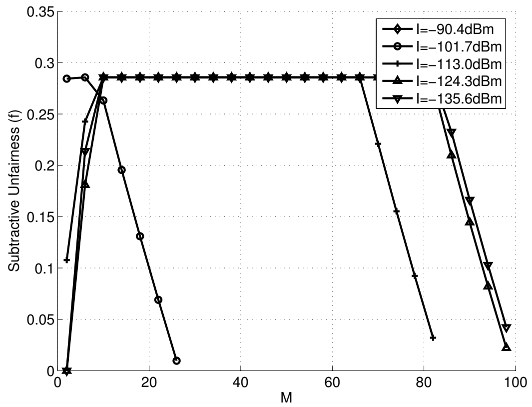

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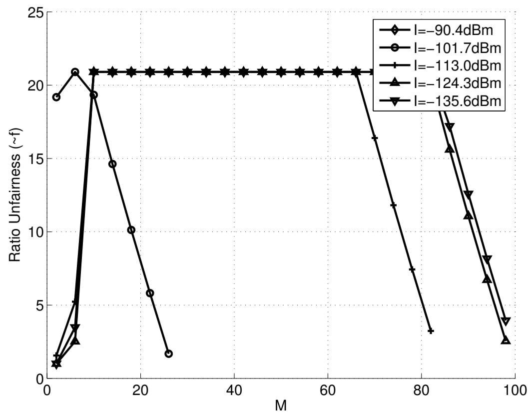
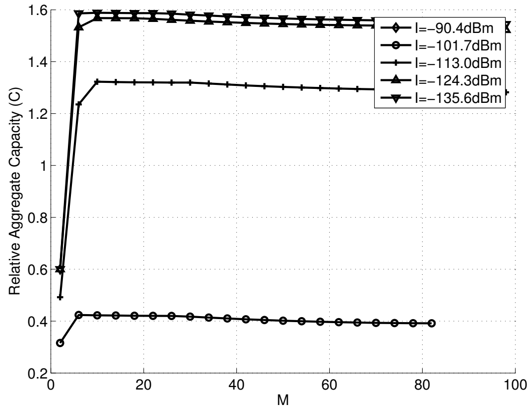


(c)

Figure 4: Results for different number of stations with varying I and $\gamma = -40dB$ in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



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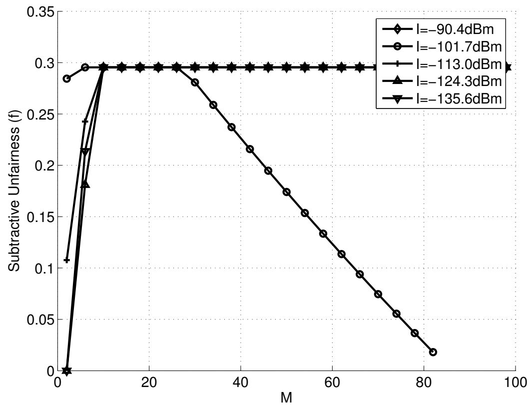

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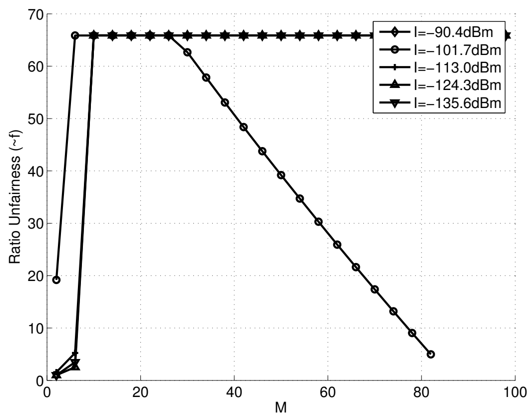


(c)

Figure 5: Results for different number of stations with varying I and $\gamma = -50dB$ in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.

when γ reduces. We argue that the high aggregate capacity of the classical problem does not actually mean that that system is actually capable of having that much revenue, because it is relying on one elite station which may not exist. The ratio unfairness of the new problem is less than one tenth of the classical problem and the gap increases as γ reduces, because the classical problems becomes more partial.

Figure 6 shows the case when γ is zero, means there is no minimum guaranteed quality of service. In this situation the ratio unfairness increases to more than ten thousand, still one thirtieth of the classical case [7]. Figure 7 shows the effects of varying γ. Here, we see that having $\gamma > -80dB$ the ratio unfairness becomes acceptable. Also, note that the number of stations has no effect on either subtractive or ratio unfairness curves. Figure 8 shows how varying I affects the solution. Note how increasing I over $-130dB$ drops the aggregate capacity curves to great extents. Again, compared to the classical case the unfairness curves are independent of the number of stations [7]. Figures 9 and 10 shows the effects of p_{max} and P_{max} on the solution. Finally, Figure 11 shows that decreasing η decreases the aggregate capacity, because the maximum capacity of each station is then limited. However, as expected, reducing η decreases the unfairness.

To have a better insight about the new problem and its benefits over the classical problem an experiment was carried out to compare their behavior in the long-run. We assume that M stations are in a cell. Here $M = 5$ is selected to make a better visualization. Each station is first randomly put in the cell. We assume that each station is a pedestrian. Hence, the movements of each station is modeled as a random walk. Denoting the position of the i–th station at time t as $\vec{\mathbf{x}}_i(t)$ we calculate, $\vec{\mathbf{x}}_i(t+dt) = \vec{\mathbf{x}}_i(t) + s\left[\cos\theta \quad \sin\theta\right]^T vdt$. Here, θ is a uniform random variable in the interval $[0, 2\pi]$ and v equals $5Km/h$ [10]. Also, s is a uniform random variable in the interval $[0, 1]$. Here, we assume that no stations leaves the cell. Also, we do not

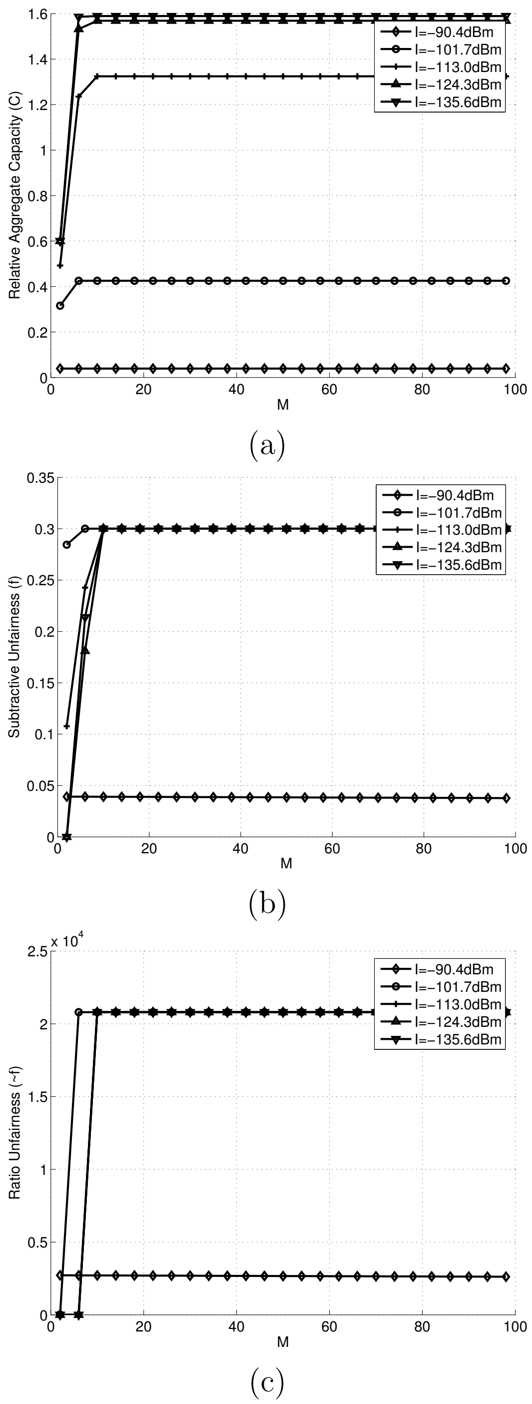
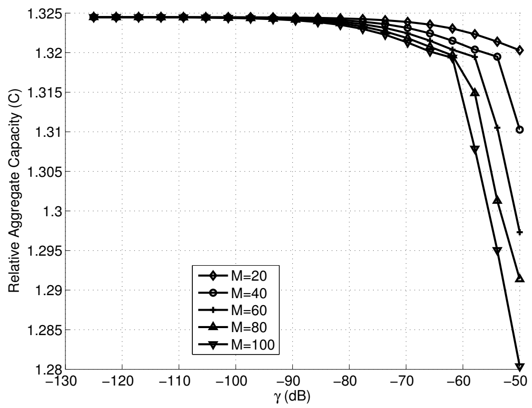


Figure 6: Results for different number of stations with varying I when γ equals zero in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



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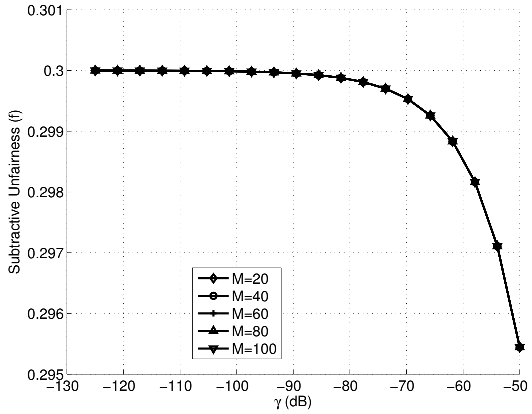

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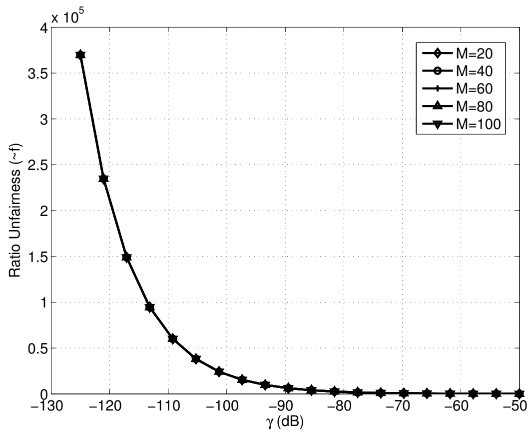

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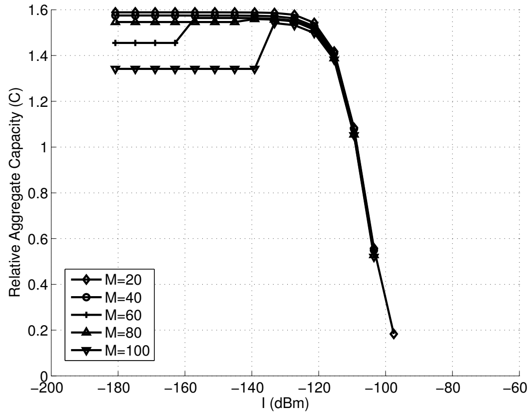
Figure 7: Results for different number of stations with varying γ in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



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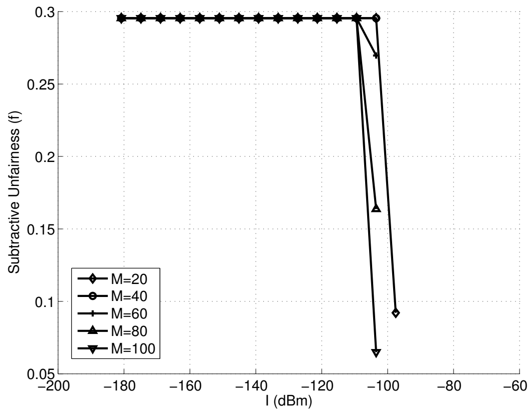

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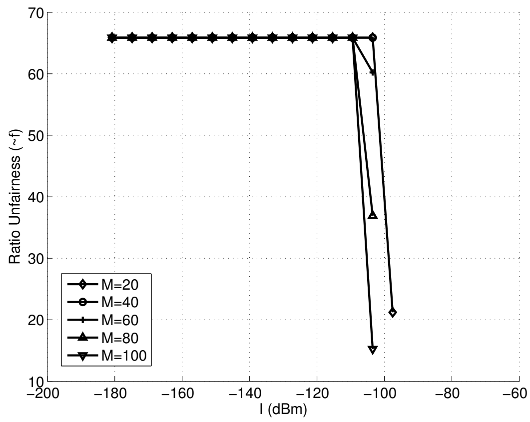

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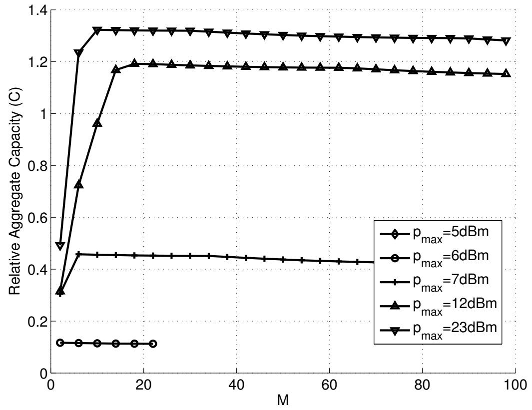
Figure 8: Results for different number of stations with varying I in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



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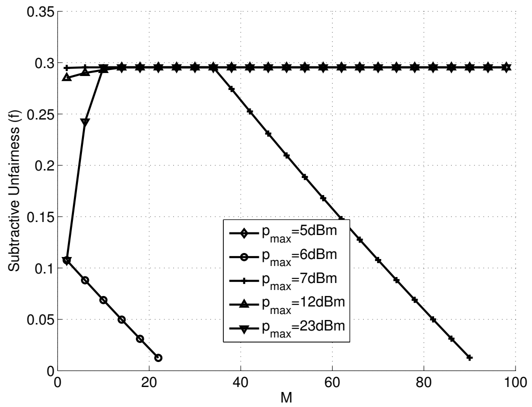

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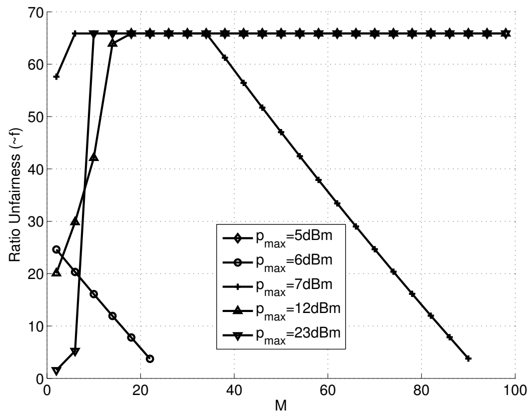

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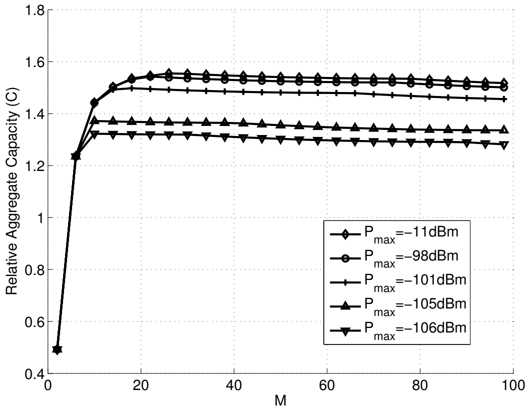
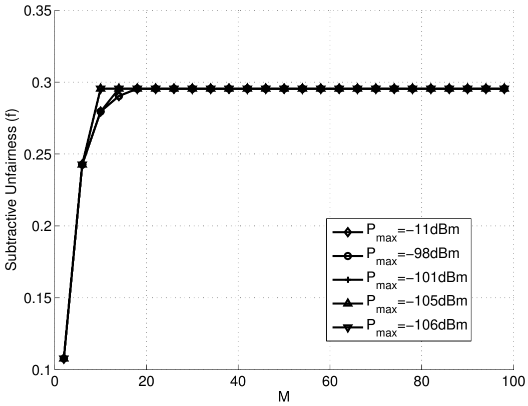
Figure 9: Results for different number of stations with varying p_{max} in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



(a)



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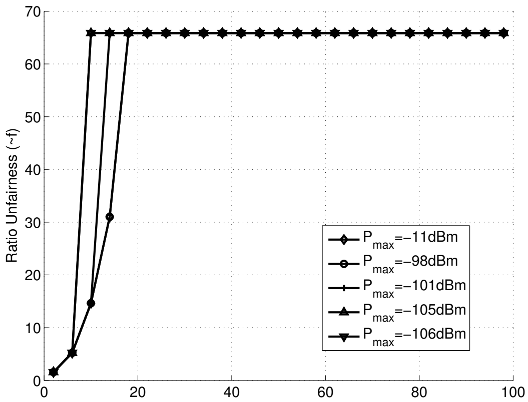

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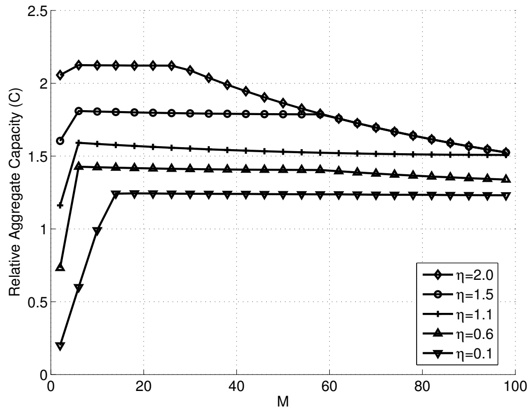
Figure 10: Results for different number of stations with varying P_{max} in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



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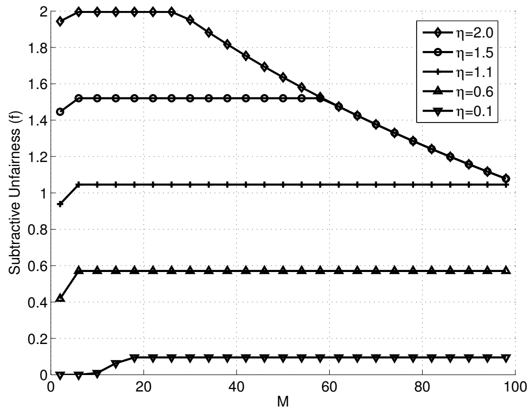

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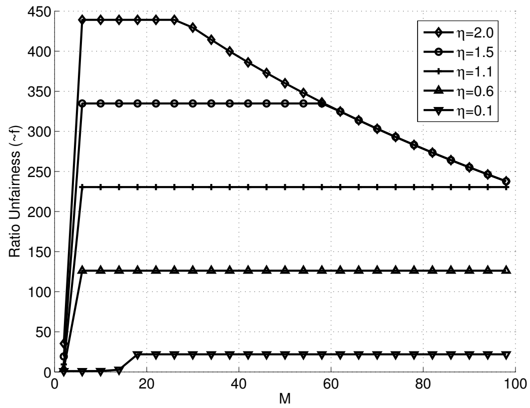
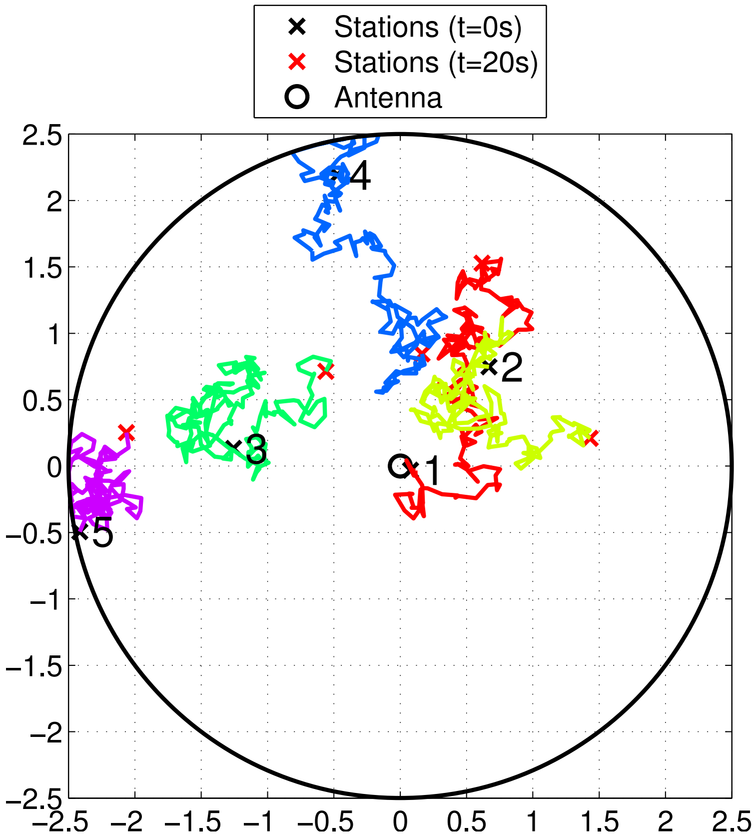


(c)

Figure 11: Results for different number of stations with varying η in the new problem. (a) Aggregate Relative Capacity. (b) Subtractive Unfairness. (c) Ratio Unfairness.



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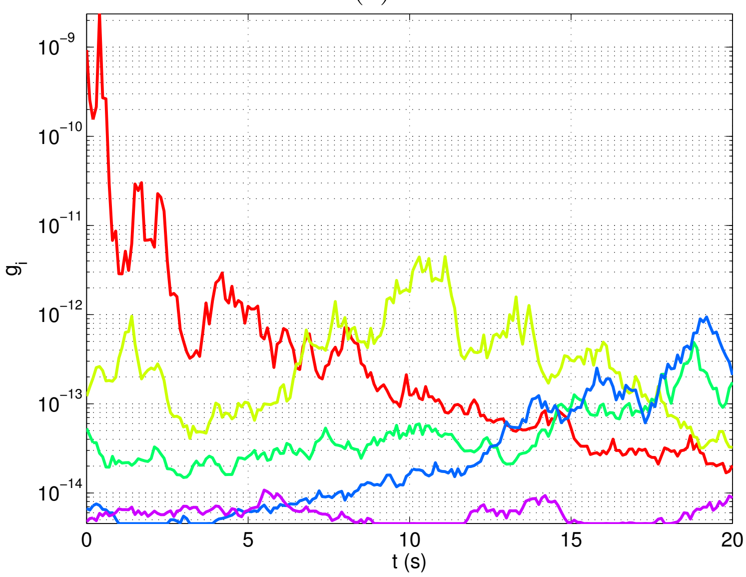


(b)

Figure 12: (a) Pattern of movement of stations used in the simulation. (b) Values of gain for different stations over time.

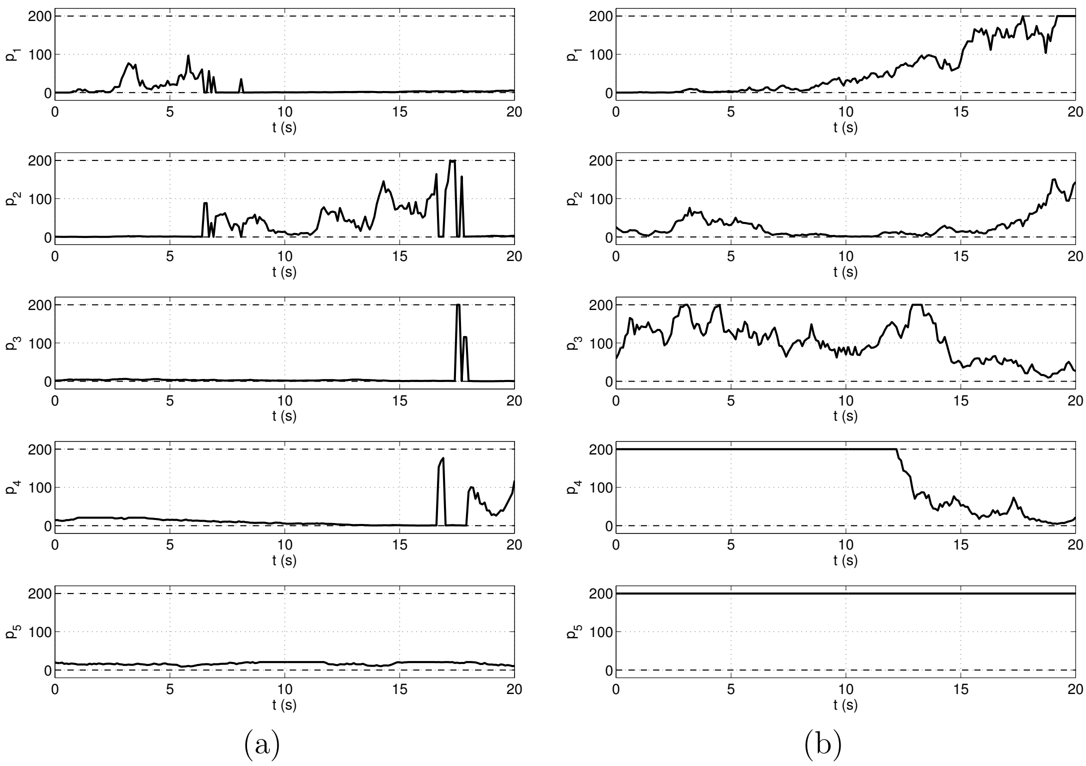
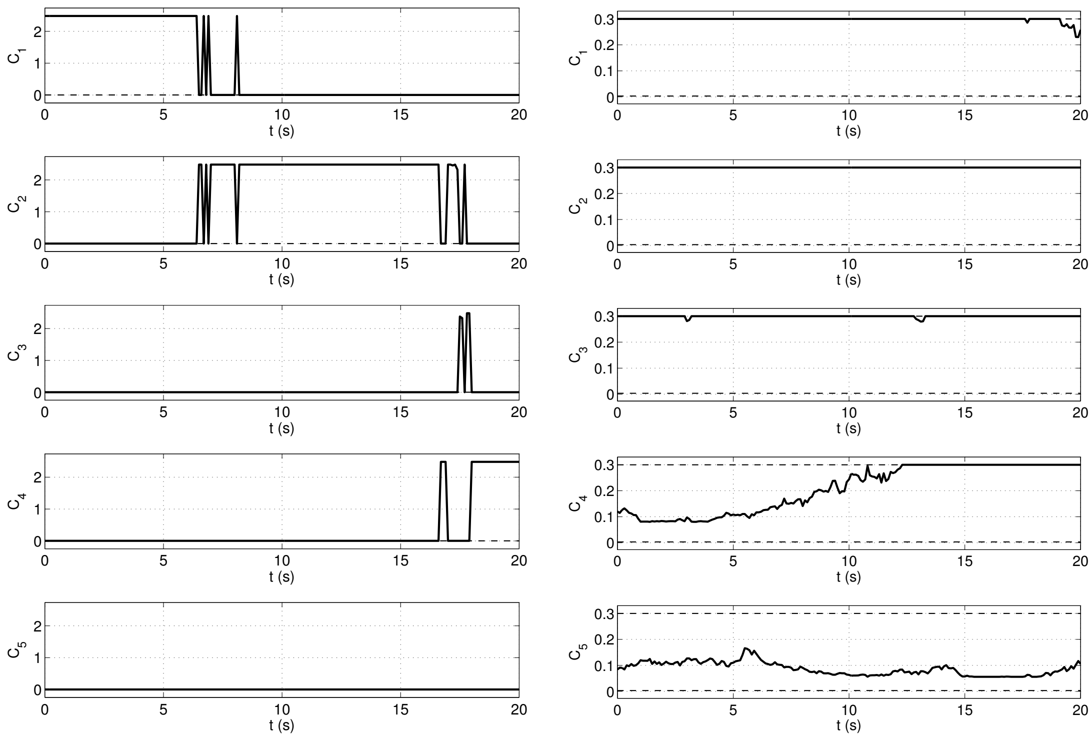


Figure 13: Transmission power of different stations over time. (a) Classical problem. (b) New problem.

consider the ones that enter it. Hence, at each time epoch we normalize the vectors $\vec{\mathbf{x}}_i(t)$ which leave the cell as, $R\,\|\vec{\mathbf{x}}_i(t)\|^{-1}\,\vec{\mathbf{x}}_i(t)$. In this way, the boundaries of the cell are assumed to reflect stations. Now, selecting values of $T = 20s$ and $dt = 0.1s$ and generating sequences of the random variables θ and s we produce a pattern of movement of the stations as seen in Figure 12–a. This pattern of movement results in different values of g_i over time (see Figure 12–b). Clearly, as a station gets far from the antenna the respective value of g_i decreases and vice versa. Now, we analyze the result of solving the classical and the new problems in this situation. First note that while the system is sampled every $100ms$ the solutions take only $7ms$ and $36ms$ for the two problems, respectively. This means the system is only being utilized 7% and 36% of the time for the classical and the new problems, respectively.



(a) (b)

Figure 14: Capacity of different stations over time. (a) Classical problem. (b) New problem.

Figure 13 compares the pattern of transmission power of different stations over time. Comparison shows that the classical problem tends more to force stations to have rapid increase and decreases in the power. That's mainly because the classical problem tries to devote almost all resources to the station which is the closest to the base. As the stations move around they compete for this occasion. Figure 14 shows the resulting capacities of different cells over time. The curves for the classical problem show that the stations are mostly oscillating between two situations of minimum capacity and a very high one. This is another clue into the classical problem's tendency to serve one station with a very high capacity. On the other hand, the new problem keeps the capacity of different cells between the two specified bounds. The interesting part is that the capacity of none of the stations is ever squeezed down to the minimum guaranteed limit (this may not always be true). Also, see how less oscillating the capacities in this situation are.

To better understand the difference between the two problems we compare the values of capacity share of different stations in the two solutions. As the capacity shares of the stations for each problem at any moment sum to one we show their values using bar graphs (see Figure 15). As anticipated, the classical problem tends to produce its revenue from one station at any moment. We can see in Figure 15–(a) that almost all bandwidth at each moment is devoted to the closest station (compare Figure 15–(a) with figures 12–b). In contrary, the new problem only increases its dependency upon each station as it gets closer. In this way while the system is looking for more possible revenue is it not demanding stations to transmit at bandwidths which are out of the range determined by γ and η. Also, no station is ever left with the minimum guaranteed capacity. Though, Figure 14–a shows that the classical problem very frequently serves stations at the lowest possible capacity.

Comparing Figures 16–(a1) and 16–(a2) we find out that the interesting results

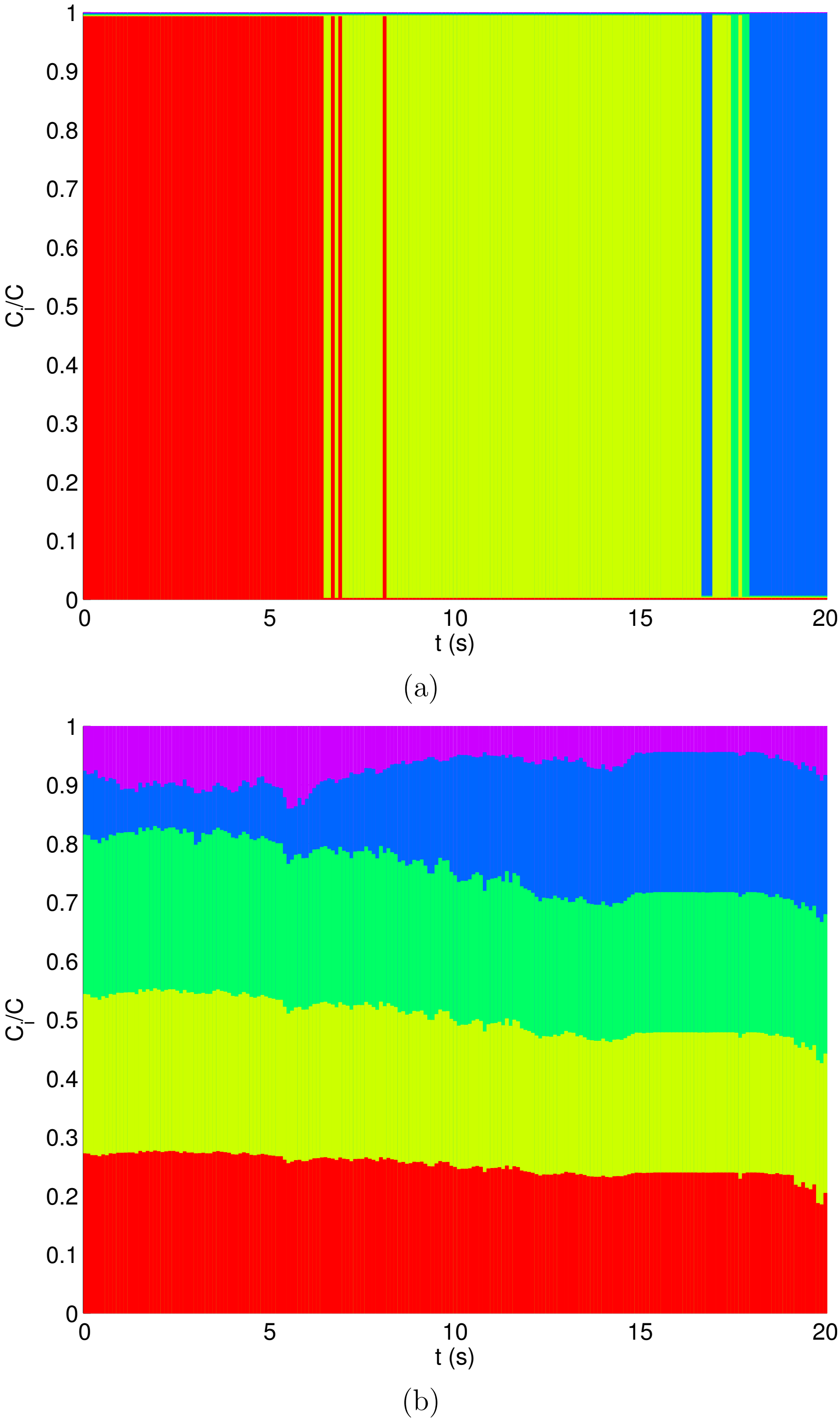
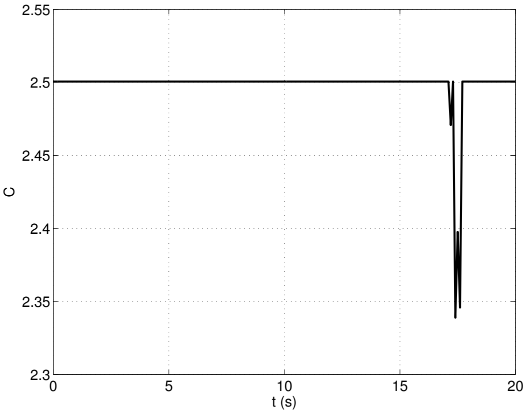
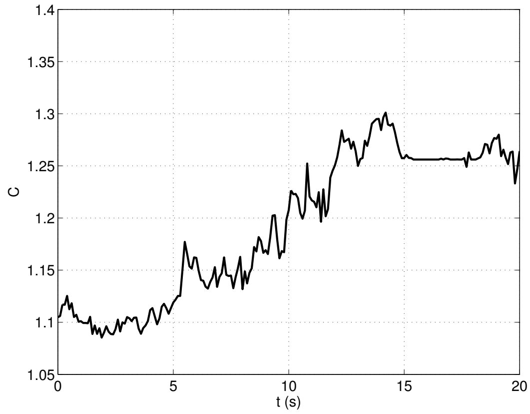


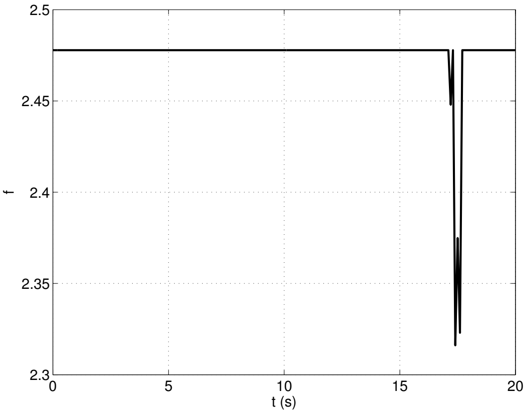
Figure 15: Capacity share of different stations over time. (a) Classical problem. (b) New problem.



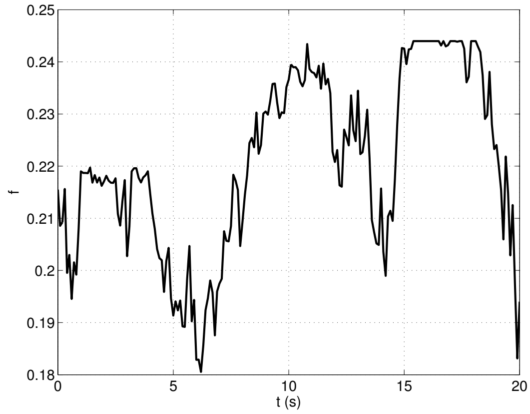
(a1)



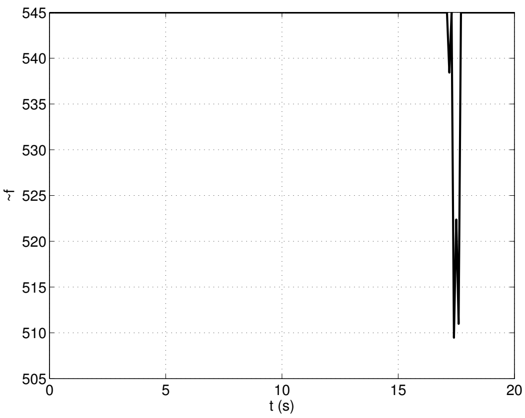
(a2)



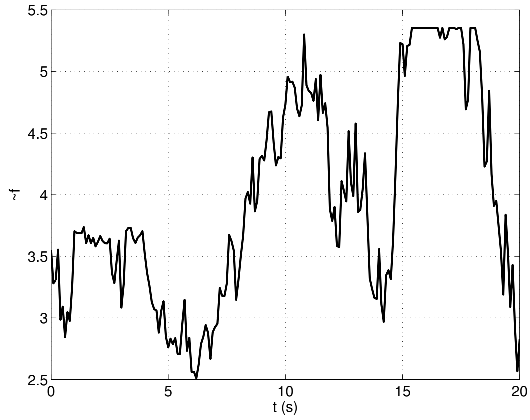
(b1)



(b2)



(c1)



(c2)

Figure 16: Aggregate capacity and unfairness of the solutions over time. (a1), (b1), and (c1), Classical problem. (a2), (b2), and (c2), New problem. (a1) and (a2) Aggregate Relative Capacity. (b1) and (b2), Subtractive Unfairness. (c1) and (c2), Ratio Unfairness.

of the new problem have the cost of having almost half aggregate capacity. On the other hand, comparing Figures 16–(b1) and 16–(c1) with Figures 16–(b2) and 16–(c2) shows that the new problem is massively more fair (almost one hundred time more in ratio). Given this experiment and the other results presented in this paper we conclude that the new problem leads to a more practical solution to the QoS problem compared to the classical formulation.

4 Conclusions

In this paper we showed that adding a maximum capacity constraint to the classical QoS problem puts limits on the unfairness of the system. Also, it was shown that this constraint inly increases the computational cost from $O(M^2)$ to $O(M^3)$. The mathematics of the new problem were carefully analyzed and using the tools and concepts developed for the classical problem an algorithm for solving the new problem was proposed and developed. Then, based on extensive experimental results the performance of the new system was analyzed and also compared with the classical one. To give a better insight into the new problem and its benefits over the classical one two scenarios were discussed. First, using the same set of parameters, plus the new maximum bound for the proposed problem, the solutions given by the two problems were discussed. It was observed that in accordance to other evidence the classical problem demands one station to transmit at a very high rate. Then, a dynamic case was investigated. The experiment revealed that the classical problem separates the set of the stations into all but one which are served at minimum possible rate and one elite, the closest station, which is served with most possible capacity, allowed by the constraints. In contrary, the new problem was observed to distribute resources evenly between stations and also to reasonably increase reliance on stations when they get closer. Based on all evidence it was suggested that the application of the new problem leads to a more practical solution with an affordable increase in the computational cost.

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