A New FPCA–Based Fast Segmentation Method for Color Images

Arash Abadpour [1] and Shohreh Kasaei [2]
[1] Mathematics Science Department, Sharif Univ. of Tech., Tehran, Iran, email: abadpour@math.sharif.edu
[2] Computer Engineering Department, Sharif Univ. of Tech., Tehran, Iran, email: skasaei@sharif.edu

***Abstract*— Fuzzy objective function–based clustering methods are proved to be fast tools for classification and segmentation purposes. Unfortunately, most of the available fuzzy clustering methods are using the spherical or ellipsoidal distances, which are proved to result in spurious clusters, when working on color data. In this paper, a general case of clustering is discussed and a general method is proposed and its convergence is proved. Also, it is proved that the FCM and the FCV methods are special cases of the proposed method. Based on the general method, a special case for color image processing is proposed. The clustering method is based on a likelihood measure, and is proved to outperform the Euclidean and the Mahalanobis distances, in color fields. Based on the proposed color clustering method, a new fast fuzzy segmentation method is proposed and is proved to be highly efficient. Comparison of the results with the FCM, proves the superiority of the proposed segmentation method.**

I. Introduction

As a non–hierarchical clustering method, fuzzy clustering, has proved to be efficient in clustering a set of given vectors into a few homogenous groups [1]. The fuzzy clustering is becoming more popular because it produces the *crisp* results when needed [2]. Also, fuzzy clustering is less prone to falling into local optima than the crisp clustering algorithms [3].

The idea of fuzzy clustering came from the *Hard C-Means* (HCM) method proposed by *Ruspini* (1969) [4]. *Dunn* (1973) [5] generalized the minimum–variance clustering procedure to a Fuzzy *ISODATA* clustering technique. *Bezdek* (1981) [6] generalized *Dunn*'s approach, by defining the fuzziness concept, and proposed the *Fuzzy C-Means*(FCM) algorithm. An extension to the FCM is the *Gustafson–Kessel*(GK) method [7], which uses the covariance matrix of the data to capture ellipsoidal properties of the clusters (*Mahalanobis* distance). After that, the *Gath–Geva*(GG) method used the same distance [8]. Other contributions in this field, include the *fuzzy c–varieties*(FCV) and *fuzzy c–elliptotypes*(FCE) [9], fuzzy c–spherical shells [10], clustering algorithms based on volume criteria [11] and fuzzy and possibilistic shell clustering [12]. In 1993, *Krishnapuram* and *Keller* proposed the *probabilistic fuzzy C–means*(PCM) clustering method [13]. Although, PCM adds more noise robustness to the FCM, but it uses the same definition of the Euclidean distance between the points and the clusters.

Any clustering method is based on the membership values, computed in terms of a distance function [1]. Although, the color clustering is an inherently ambiguous task, because of the edge blurring [3], but the importance of choosing a proper distance function is overlooked in the color image processing literature. For example, many authors have used the *Euclidean* distance based homogeneity criteria, in the color domain, with no explicit proof of its performance [14], [15], [16], [17], [18]. It is proved that the *linear partial reconstruction error*(LPRE), results in a proper likelihood measure for processing natural color images [19]. In this methodology, the likelihood of the vector $\vec{c}$ to the cluster r is defined as $e_r(\vec{c}) = \|\acute{\vec{v}}(\vec{c}-\vec{\eta})\vec{v} - (\vec{c}-\vec{\eta})\|$, where, $\vec{v}$ shows the direction of the first principal component and $\|\vec{x}\|$ denotes the normalized L_1 norm. The comparison of the LPRE with the conventional *Euclidean* and *Mahalanobis* distances, has proved its superiority, both in terms of likelihood measurement and homogeneity decision [20]. In fact, the Euclidean and the Mahalanobis distances lead to spurious likelihood and homogeneity decisions in color fields [20]. Thus, the FCM, the HCM, and the PCM, (which are based on the Euclidean distance measurement), are theoretically inappropriate for color clustering. The same happens for the GK, GG, and FEC methods, (which use the Mahalanobis distance).

The idea of reducing the color space dimension is not a new idea; many researchers have reported benefits of illumination coordinate rejection [21]. As a quadratic dimension reduction tools, the *principle component analysis* (*PCA*) [22] is widely used in signal processing, statistics, and neural networks. The fuzzy extension of the PCA is not wide spread in the literature. In [23] the authors proposed an Euclidean distance based definition for the fuzzy covariance matrix. In [24] the authors embed the fuzziness idea to the definition of what they call the *scatter* matrix. Here we propose and prove a unifying definition for the FPCA.

II. Proposed Method

A. Fuzzy Principal Component Analysis(FPCA)

Assume, performing the PCA transform on the set of vectors $\vec{x}_i, i = 1\cdots N$, while the samples are members of the discrete field $\Phi = \{\vec{\phi}_i | i = 1\cdots n\}$. This situation happens when we are working on color vectors, which are repetitions of the vectors available in $\Phi = (N_{255}\cup\{0\})^3$. Hence, we are facing the PCA problem for the set of fuzzy vectors $\{(\vec{\phi}_i; p_i) | i = 1\cdots n\}$, where p_i equals the number of repetitions of $\vec{\omega}_i$ (the histogram). Assume the general problem of finding the principal components of $\{\vec{x}_i; p_i | i = 1\cdots n\}$, when not restricting $\sum_{i=1}^n p_i = 1$. Here, as we are only concerned with the zero and one–dimensional representation of the data cloud, we will derive the formulation for $\vec{\eta}$ and $\vec{\omega}_1$ (but the same method leads to the computation of other principal components as well). All objective functions in the PCA theory are of the form, $\Delta(\vec{x}_\circ) = \sum_{i=1}^n Q_{\vec{x}_\circ}(\vec{x}_i)$, where, $Q_{\vec{x}_\circ}(\vec{x}_i)$ is a quadratic function. Assuming the case of repeated vectors, the objective function changes to $\Delta(\vec{x}_\circ) = \sum_{i=1}^n p_i Q_{\vec{x}_\circ}(\vec{\phi}_i)$. Thus, it is reasonable to define the objective functions in the fuzzy domain as, $\Delta(\vec{x}_\circ) = \sum_{i=1}^n p_i Q_{\vec{x}_\circ}(\vec{x}_i)$, too. Note that the case of $p_i \equiv 1$ leads to the same non–fuzzy definition. Also, the assumption of $\sum_{i=1}^n p_i = 1$ is not important, because it shows itself as a constant scaling factor, (not affecting the minima). The objective function for the expectation vector in the crisp domain is defined as $Q_{\vec{x}_\circ}(\vec{x}) = \|\vec{x} - \vec{x}_\circ\|^2$ [22]. Thus, the fuzzy expectation is the minima of the objective function defined as $\Delta_\circ(\vec{x}_\circ) = \sum_{i=1}^n p_i \|\vec{x} - \vec{x}_\circ\|^2$. Assigning the derivative of $\Delta_\circ(\vec{x}_\circ)$ in terms of $\vec{x}_\circ$ to zero, results in $\vec{\eta} = \Sigma_{i=1}^n p_i \vec{x}_i / \Sigma_{i=1}^n p_i$. The objective function for the crisp first principal direction is the direction of the maximum deviation or equivalently the direction of minimum one–dimensional reconstruction error, defined as $Q_{\vec{x}_\circ}(\vec{x}) = \|\vec{x} - \vec{\eta} - \vec{x}_\circ'(\vec{x} - \vec{\eta})\vec{x}_\circ\|^2$ [22]. Thus in the fuzzy domain, we should minimize the objective function defined as, $\Delta_I(\vec{x}_\circ) = \sum_{i=1}^n p_i \|\vec{x} - \vec{\eta} - \vec{x}_\circ'(\vec{x} - \vec{\eta})\vec{x}_\circ\|^2$. Algebraic derivation of the objective function and incorporating the size constraint on the principal components, ($\|\vec{x}_\circ\|^2 = 1$), leads to $\Delta_I(\vec{x}_\circ) = \sum_{i=1}^n p_i \|\vec{x}_i - \vec{\eta}\|^2 - \vec{x}_\circ' \tilde{C} \vec{x}_\circ$, where, $\tilde{C}$ is the *fuzzy covariance matrix* defined as $\sum_{i=1}^n p_i (\vec{x}_i - \vec{\eta})(\vec{x}_i - \vec{\eta})'$. As the first term in the objective function, ($\sum_{i=1}^n p_i \|\vec{x} - \vec{\eta}\|^2$), is not a function of $\vec{x}_\circ$, thus, $\Delta_I(\vec{x}_\circ)$ is minimized, when the second term, ($\vec{x}_\circ' \tilde{C} \vec{x}_\circ$), is maximized. Using the method of *Lagrange* multipliers for embedding the $\|\vec{x}_\circ\| = 1$ constraint, we should maximize the objective function defined as $\bar{\Delta}_I(\vec{x}_\circ) = \vec{x}_\circ' \tilde{C} \vec{x}_\circ + \lambda(\|\vec{x}_\circ\|^2 - 1)$. Differentiating the new objective function in terms of $\vec{x}_\circ$ and assigning the result to zero, we have $\tilde{C}\vec{x}_\circ = \lambda \vec{x}_\circ$. Thus, $\vec{x}_\circ$ is an eigenvector of $\tilde{C}$, resulting in $\tilde{\Delta}_I(\vec{x}_\circ) = \lambda$. Thus, the direction of the first fuzzy principal component is the eigenvector of $\tilde{C}$, corresponding to the largest eigenvalue. It can be easily proved, (using the same method), that other principal directions correspond to other eigenvectors of $\tilde{C}$ sorted by the eigenvalues in a descending fashion.

B. Weighted Powered Sum Minimization

Assume the optimization problem to be defined as minimizing the function $\Delta(x_1 \cdots x_n) = \sum_{i=1}^n x_i^m w_i$ under the assumption of $\sum_{i=1}^n x_i = 1$. Replacing $x_p = 1 - \sum_{i=1, i\neq p}^n x_i$ in the objective function and differentiating with respect to x_i for $i \neq p$ results in $x_p = w_p^{-\frac{1}{m-1}} / \Sigma_{i=1}^n w_i^{-\frac{1}{m-1}}$. The normality constraint is satisfied. Putting the result in the objective function and and assuming $m > 1$, it is obvious that $\forall p, \Delta \leq w_p$. Also, note that the case of setting all x_i equal to zero, except for $x_p = 1$, leads to $\Delta = w_p$. Hence, having in mind that the gradient of Δ gets zero just once, while the value of Δ at that point is smaller than some marginal points, the acquired result leads to the global minima of the problem in hand.

C. General Clustering Algorithm

Assume the general clustering problem stated as minimizing the objective function defined as $J(X, \Phi) = \sum_{i=1}^n \sum_{j=1}^c p_{ij}^m D_{ij}$, which describes the best choice of clustering the data points $X = \{\vec{x}_1 \cdots \vec{x}_n\}$ into c clusters described by $\Phi = \{\phi_1 \cdots \phi_c\}$. Here, p_{ij} is the fuzzy membership of $\vec{x}_i$ to the jth cluster and D_{ij} is the distance between this point and the cluster. Assume that $D_{ij} = \Psi(\vec{x}_i, \phi_j)$ is the *appropriate* distance function for the vector geometry under investigation. Note that under the Ψ distance function, we have $p_{ij} \propto Dij$ and $p_{ij} \propto D_{pj}^{-1}$ for $p = 1 \cdots n, p \neq i$. Here, ϕ_j is the defining parameters of the jth cluster according to the general cluster model. Before working on the solution of the main objective function, we will discuss a special case. The FCM, models each cluster with a single vector ($\phi_j = \{\vec{\eta}_j\}$), defining the Ψ function as the squared Euclidean distance between the given vector and the cluster center. The key point of the proposed *general fuzzy clustering*(GFC) algorithm is the deep relation between the cluster model and the cluster tuning function. Note that for a set of given points, the expectation vector, minimizes the sum of the squared Euclidean distances. Assume that the function Υ tunes the cluster model ϕ_j to best fit the points, meaning that for the fuzzy set $\tilde{X} = \{(\vec{x}_i; p_i) | i = 1 \cdots n\}$ of vectors, $\phi = \Upsilon(\tilde{X})$ is the solution for $\phi^* = \arg_\phi \min\{\Sigma p_i \Psi(\vec{x}_i, \phi)\}$. Then, $\Upsilon(\tilde{X}) = \tilde{E}\{\tilde{X}\}$ is the solution for $\Psi(\vec{x}, \vec{\eta}) = \|\vec{x} - \vec{\eta}\|^2$. Here, $\tilde{E}\{\tilde{X}\}$ stands for the fuzzy expectation of a fuzzy set. Turning back to the main problem, assume that we have the dual functions $\Psi(\cdot)$ and $\Upsilon(\cdot)$. We propose an algorithm that converges to a minimal point of the main objective function, if at least one exists. Now, rewrite the objective function as $J(X, \Phi) = \sum_{i=1}^n \Delta_i$, with $\Delta_i = \sum_{j=1}^c p_{ij}^m D_{ij}$. Also, assume that we have fixed D_{ij} and we are trying to decline $J(X, \Phi)$ by working on p_{ij}. As the only restriction on p_{ij} is the normality condition ($\forall i, \sum_{j=1}^c p_{ij} = 1$), for $i \neq i'$ there is no connection between p_{ij} and $p_{i'j}$. Thus, declining Δ_is independently, results in declination of $J(X, \Phi)$. Having in mind the normality constraints, minimizing Δ_i is a weighted powered sum minimization, discussed in Section II-B. Hence, the result is $p_{ij} = D_{ij}^{-\frac{1}{m-1}} / \sum_{k=1}^c D_{ik}^{-\frac{1}{m-1}}$. Note that, during this stage, $J(X, \Phi)$ is declined, except for the case that p_{ij}s are satisfying the equation at first. We will come back to this situation later. Now, assume rewriting the objective function as $J(X, \Phi) = \sum_{j=1}^c \Theta_j$. Here, $\Theta_j = \sum_{i=1}^n p_{ij}^m D_{ij}$. Assume that we have fixed p_{ij} and we are trying to decline $J(X, \Phi)$ by working on D_{ij}. Declining Θ_j means tuning the jth cluster to minimize the overall distances in a fuzzy scheme that can be solved independently for different clusters, resulting in overall decline of the $J(X, \Phi)$. The solution in this state is clearly obtained by using the Υ function as $\phi_j = \Upsilon(\{(\vec{x}_i, p_{ij}^m) | i = 1 \cdots n\})$. Again, the $J(X, \Phi)$ can never increase in this state. (Its constancy shows that all the clusters have been in the best place, according to the special distance function Ψ.) In the proposed clustering algorithm, these two consecutive stages are repeated, while the stationarity condition is met: (a) compute p_{ij} values according to the Ψ function and (b) tune the clusters according to the Υ function. Note that during these two stages, $J(X, \Phi)$ never rises and the case that it remains constant in two consecutive stages, results in its being constant for all coming stages. Thus, using this method, $J(X, \Phi)$ is going towards a minimum point, and never gets oscillated. The halting test of the algorithm is easily derived in terms of the cluster parameters, not changing. Note that rather than FCM, other methods like GK, FEC, and FCV, are special cases of the proposed general clustering method.

D. Color Clustering

As stated in Section I, the LPRE distance defined as $\Psi(\vec{c}, [\vec{\eta}, \vec{v}]) = \|(\vec{c} - \vec{\eta}) - \vec{v}'(\vec{c} - \vec{\eta})\vec{v}\|^2$, results in a good subjective clustering of color vectors. In this approach the cluster model is a *cylinder* with the central axis having $\vec{\eta}$ and parallel with $\vec{v}$. Also, the dual function $\Upsilon(\tilde{X})$ computes the fuzzy expectation and the first fuzzy principal component of $\tilde{X}$ as new values of $\vec{\eta}$ and $\vec{v}$, as discussed in Section II-A. It must be emphasized that this formulation results in a special case of the FCV method setting $r = 1$ [9]. We propose putting a threshold (ε) on the changes of the coordinates of cluster centers as the halting condition, as used generally in the fuzzy clustering theory [1].

E. Color Segmentation

Clustering the image I, into c clusters using the method proposed above results in a set of c images $J_i, i = 1 \cdots c$ which show the likelihood of each pixel to the each of the clusters. Note that J_i satisfies $\forall x, y : \sum_{i=1}^{n} J_i(x, y) = 1$. Now, assume a $p \times p$ smoothing convolution kernel M. It is clear that M satisfies $\sum_{i=1}^{p} \sum_{i=1}^{p} M_{ij} = 1$. Hence, Applying M to each J_i independently to acquire $\tilde{J}_i$, the new membership maps, also satisfy the normality condition. Thus, $\tilde{J}_i, i = 1 \cdots c$ can be assumed as the smoothed likelihood to the clusters. The main benefit of using $\tilde{J}_i$ over J_i is the smoother resulting segments. The crisp segmentation result is obtained using the maximum likelihood and we propose to use a simple averaging kernel for smoothing.

III. Experimental Results

The tests are performed using a PIII 1600MHz personal computer with 256MB of RAM on a large digital image archive, containing professional photographs and standard images. All samples are medium sized (512×512), high–quality JPEG images in RGB format. Figure 1 shows some of the test images.

Figure 2 shows the results of the proposed segmentation method with the c parameter shown in Table I. In this table, t_1 shows the time elapsed by the proposed method, while t_2 shows the time elapsed by the conventional FCM. Here, the radius of the convolution kernel is selected to be 5. Investigating Table I, the low computational cost of the proposed method is clear. Figure 3 shows the segmentation results produced by the conventional FCM. Comparing the results of the proposed segmentation method and the FCM, reveals the performance of the proposed method. While for Figures 1-a, 1-b, and 1-c, the results of the two methods are almost the same, both in the spatial domain and the spectral domain, considering the results on Figure 1-d is important. The FCM has failed to distinguish between the two red and yellow cloths, because of their close spectral zones. Also, in the Figure 1-e, the FCM has classified the apple and the cucumber in the same group and in the case of Figure 1-g, the FCM has gathered some parts of the chimney and the red ribbon in the same group, also the grass and some parts of the sand. In Figure 1-h, the FCM has failed to separate the points belonging to the wall and the kerchief, while in all these cases the proposed method has resulted perfect. In Figures 1-i and 1-j, there are mistakes in the segmentation results of the FCM, in distinguishing the face and the cloth and classifying the blue shades of the sky in the windows and the balcony ceils, respectively. In Figure 1-k the FCM has not been able to partition the flowers completely, compared to the perfect results of the proposed method. Also, investigate the poor segmentation results of the salad image in Figure 1-l, in which the carrot and the vegetables are put in the same class. Table I shows that the proposed method is most of the times faster than the FCM.

IV. Conclusions

Although, researchers generally use the Euclidean and the Mahalanobis–based clustering and segmentation methods, in this paper we proved that the color clusters in typical images are neither spherical, nor ellipsoidal in all 12 tested standard color spaces. A new general fuzzy clustering method is proposed for arbitrary shapes of clusters and its convergence is proved mathematically. It is proved that the well–known FCM clustering method is a special case of the proposed method. Also, a new fuzzy clustering method is proposed for color images and it is proved to be highly efficient. Also, its subjective perception is shown to be satisfactory. Based on the proposed clustering method, a new fast and efficient segmentation method is proposed and its performance is evaluated.Tthe performance analysis comparison of the proposed segmentation method and the FCM, proved the superiority of the FCM in color fields.

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TABLE I

SEGMENTATION PARAMETER AND ELAPSED TIME. [m: FUZZYNESS. t_1: PROPOSED METHOD. t_2: FCM.]

Sample	1-a	1-b	1-c	1-d	1-e	1-f	1-g	1-h	1-i	1-j	1-k	1-l
c	3	3	4	4	4	3	4	3	4	3	3	4
$t_1(s)$	12	8	11	7	12	3	16	5	8	3	3	17
$t_2(s)$	7	9	14	9	20	4	16	13	5	9	6	15

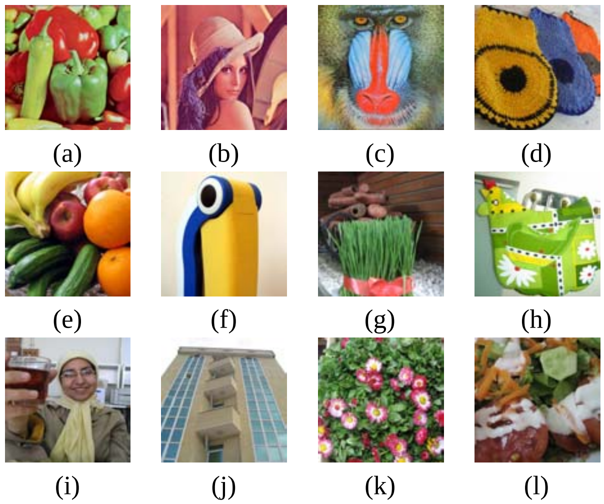

Fig. 1. Some typical test images, (a)–(c) Standard images, (d)–(l) Digital Color Image Archive, *Shohreh Tabatabaii Seifi* and *Ali Qanavati*, *Qnavati@mehr.sharif.edu*.

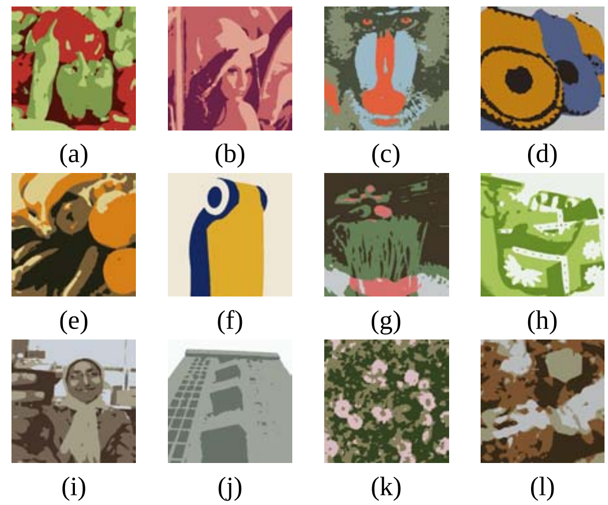

Fig. 3. Results of the conventional FCM method.

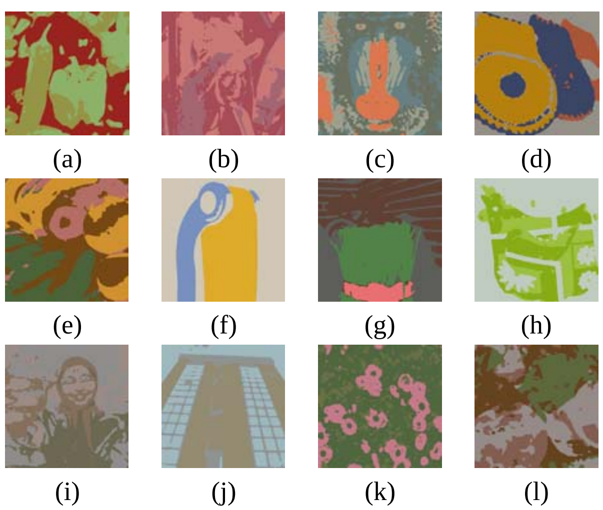

Fig. 2. Results of the proposed segmentation method.

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